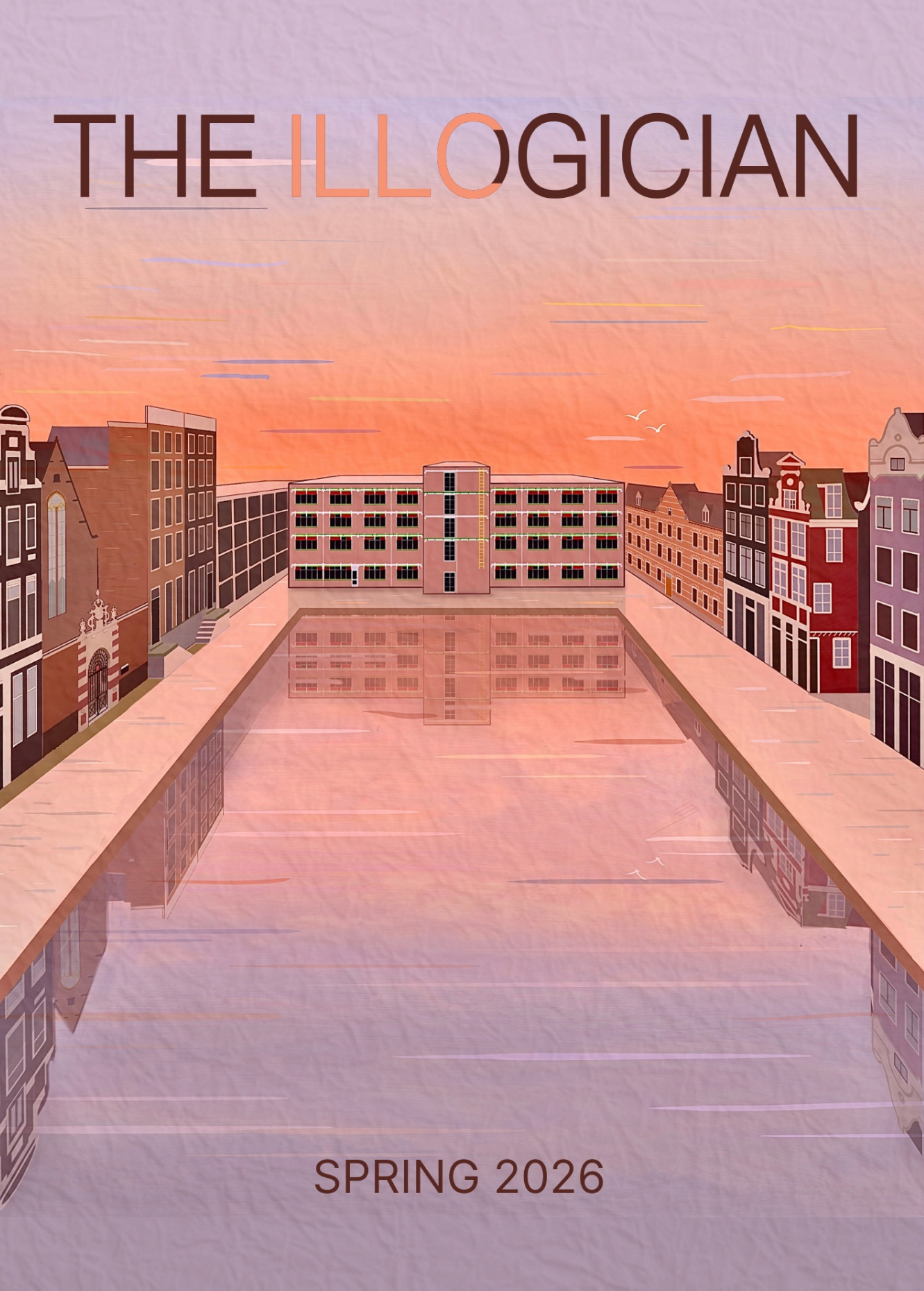
THE ILLOGICIAN



SPRING 2026

EDITORIAL

THIAGO COCCO ROQUE | EDITOR-IN-CHIEF

What started as an experiment quickly turned into something we didn't quite anticipate: a magazine that people actually read, talked about, shared, and, to our great surprise, one that even escaped the walls of the ILLC and found its way onto the UvA news page. Somewhere along the way, we learned about deadlines, layouts, editing, coordination, and the many small logistical miracles required to turn enthusiasm into a printed object.

This second edition is the result of that learning. We now know what we're doing! Well not really, but at least a little better than before.

But knowing how to do something does not mean taking it too seriously. If anything, having the technical know-how has freed us to be more playful, more daring, and more deliberate in our choices. The Illogician remains a space where curiosity comes first, where ideas can be polished or provisional, and where Logic can be treated with both respect and irreverence.

What unites these pages is not a doctrine or a methodology, but a shared delight in thinking hard, thinking weirdly, and sometimes thinking sideways.

So here is issue two: less improvised, no less imaginative. We hope you enjoy reading it as much as we enjoyed making it, and that it reminds you that even once you learn the rules, you're still allowed to play with them.

THE ILLOGICIAN

Issue 2: Spring 2026
Print Run: 160 copies

The Illogician is a semiannual, semiserious periodical specialised in the areas of logic, language, and computation. It is published by Master of Logic students at the Institute for Logic, Language, and Computation of the University of Amsterdam.

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TABLE OF CONTENTS

COMPACTNESS AND COMPLETENESS IN CONTEMPORARY ROMANTIC INTERACTION 4
THIAGO COCCO ROQUE

TOO MUCH INFORMATION? THE USEFULNESS OF TALKING REDUNDANTLY 8
THOMAS VAN DER LEER

THE STORY OF MOSCOW'S METHODOLOGICAL CIRCLE 11
MATTEO MERCURI

A CONCEPTUAL HISTORY OF BHK NEGATION 14
EDOARDO MENORELLO

PROVING EVERYTHING WHILE REVEALING NOTHING: AN INTRODUCTION TO ZERO-KNOWLEDGE PROOFS 19
MARIANA RIO COSTA

ROTATING SHAPES: TWO MÖBIUS MAKE A KLEIN 23
MAX WEHMEIER

THE LONG JOURNEY TO CRAIG INTERPOLATION FOR PROPOSITIONAL DYNAMIC LOGIC 26
JOSJE VAN DER LAAN

THE AXIOM OF BAD CHOICE AND ITS EQUIVALENTS 28
MATTEO CELLI

THE PINEAPPLE INCIDENT: A BELIEF-THEORETIC AND TOPOLOGICAL ANALYSIS 31
GIUSEPPE MANES

LOGICAL RETHORIC MAKES ILLOGICAL DEBATES 37
ALEXANDRE MAZUIR

INTERVIEW WITH PROF. CATARINA DUTILH NOVAES 40
THIAGO J. COCCO ROQUE

(IL)LOGICAL HOROSCOPE 44
MATTEO CELLI, JOSJE VAN DER LAAN, BARDO MAIENBORN, GIANNIS RACHMANIS

CRYPTIC CLUES 47
ESTEL KOOLE, DAVID KÜHNEMANN, BARDO MAIENBORN, MARIANA RIO COSTA

ILLC WORD SEARCH 48
IDSKE ROEST

COMPACTNESS AND COMPLETENESS IN CONTEMPORARY ROMANTIC INTERACTION

THIAGO J. COCCO ROQUE | PHILOSOPHY

INTRODUCTION

Attempts to formalize human preference, belief, and intention have given rise to robust epistemic and dynamic-logical frameworks. However, one domain remains resistant to logical analysis: romantic interaction.

This paper introduces *Affective Predicate Logic (APL)*, a first-order modal framework designed to capture inferential patterns arising in digital romantic communication, specifically through participant observation of the Hinge dating platform between October 2024 and October 2025. While individual affective exchanges appear logically consistent, their aggregation leads to systematic model-theoretic failure. In particular, we demonstrate violations of completeness and compactness under classical semantics. Empirical data drawn from the case study shows persistent undecidability of propositions such as *TheyLikeMe*(a)[1] and semantic collapse following interpretive divergence over "jazz."

METHODOLOGY

The study was conducted primarily through participant observation, utilizing a single-agent experiential approach over an approximate twelve-month interval. Data was collected through naturalistic interaction via the Hinge platform, supplemented by asynchronous peer commentary (sample size: up to three group chats, nine individual consultations, and unstructured café discourse). Interactions were logged chronologically and retrospectively formalized into the language $\mathscr{L}_H$, with emotional state assessments calibrated using a semi-informal Likert scale ranging from "stable confidence" to "existential doubt". Ambiguous textual and emoji-based communicative acts were analyzed under extended dynamic epistemic procedures, with updates categorized as either public announcements or *implicit epistemic state revisions*. No attempt was made to eliminate confounding variables such as self-presentation bias, algorithmic matchmaking unpredictability, or overinterpretation of punctuation. The methodology adheres to widely accepted standards of philosophical rigor, insofar as no alternative was available.

SYNTAX OF AFFECTIVE PREDICATE LOGIC

Definition (Language $\mathscr{L}_H$)

The language $\mathscr{L}_H$ consists of:

- Constants: $a_1, ..., a_{81}$ representing interlocutors.
- Unary predicates:
  - Attractive(x),
  - SuggestedCoffee(x),
  - LikedMyPrompt(x).

1. Empirical report: "conversation terminated, epistemic state unclear."

• Binary predicates:
  • Messaged(x,y),
  • AmbiguousText(x,y).
• Ternary predicate:
  • SemanticShift(x,y,p) where p is a formula or concept.
• Modal operators:
  • □φ (affectively certain)
  • ◇φ (romantically possible)
  • ♡φ (desirable but not confirmed)

We define:

• Interested(x) := ♡TheyLikeMe(x)

• Ghosted(x) := ¬∃Messaged(x, t) ∧¬□¬TheyLikeMe(x)

SEMANTICS

Definition (Affective Model)

An Affective Model is a tuple:

$M = (W, R_{aff}, R_{text}, D, I)$

where:

• W is a nonempty set of affective worlds;

• R_{aff} models transitions in emotional state;

• R_{text} models updates via message or emoji;

• D is a nonempty domain of agents;

• I assigns denotations to predicates at each world.

Evaluation is defined as usual. For example:

$$M, w \models \text{TheyLikeMe}(a) \text{ iff } (a, w) \in I(\text{LikingRelation}).$$

Importantly, truth values may update without explicit communication via *silent belief revision*, a phenomenon not permitted in classical epistemic frameworks.

INCOMPLETENESS OF THE THEORY T_H

$$T_H \vdash \varphi \text{ or } T_H \vdash \neg\varphi$$

Definition (Completeness)

The theory T_H is complete if for every closed formula φ,

Theorem (Indefinite Romantic Status)

There exists a_{42} such that

$$T_H \nvdash \text{TheyLikeMe}(a_{42}) \text{ and } T_H \nvdash \neg\text{TheyLikeMe}(a_{42}).$$

even following conversational termination.

Proof (Sketch)

Post-conversational signals (delayed reply, ambiguous emoji, “haha”) support both φ and ¬φ. Thus φ remains undecidable.

FAILURE OF COMPACTNESS

Definition (Local satisfiability)

A finite interaction set Δ ⊆ T_(H) is satisfiable iff

∃M, w such that M, w ⊨ Δ.

Observation

Each individual encounter (finite set of interactions) is locally satisfiable. However,

$$\bigcup_i \Delta_i \text{ is inconsistent.}$$

Sources of contradiction include conflicting affective commitments, overlapping timeframes, non-reconciled emoji-based belief updates, and genre-based semantic divergence (*penguin principle: all Sinatra is jazz*).

Theorem (Global Incoherence)

$$\Diamond\heartsuit\text{TheyLikeMe}(a_i) \wedge \Box\neg\text{Exclusive}(a_j) \wedge \text{AffectionallyMonogamous}(\text{subject})$$

is unsatisfiable in any Affective Model.

UPDATE LOGIC OF GHOSTING

Ghosting operates as *Silent Epistemic Collapse (SEC)*:

Before SEC: ◇♡TheyLikeMe(a)

After SEC: □¬♡TheyLikeMe(a)

with no explicit communicative act:

¬Messaged(a, t) observed as sole indicator.

CONCLUSION

It is possible to list the following logical results from this phenomena:

• a single date interaction is satisfiable

• a silent update is a non-public announcment

• a fulll affective history is inconsistent

• the definition of jazz is a semnatic collapse

• the long-term modelling is non-compact

Conjecture (Affective Incompleteness Hypothesis)

No recursively axiomatizable system captures reciprocity, mutual interest, and musical genre constraints within APL.

FUTURE WORK

Future research may involve the development of a paraconsistent logic of romance ($\mathscr{PL}$*) to manage contradictory yet emotionally active states, the application of supervaluationist semantics to model borderline flirtation scenarios (particularly those involving ambiguous textual cues such as “haha” or ellipsis), and the formulation of a temporal logic of “vibes” to represent fluctuations in affective commitment over time. Additionally, further work may incorporate dynamic epistemic frameworks to capture cases of premature exclusivity assumptions, with particular attention to non-public announcements and silent state updates. Investigating the potential integration of probabilistic semantics or quantum-style superposition models for undecided interpersonal interest remains an open direction, as does the formal treatment of genre-based inference breakdowns (notably, but not limited to, the Sinatra incident).

ILLOGICAL DATING PROFILES

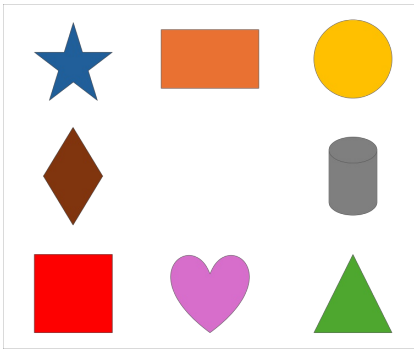
JOSJE VAN DER LAAN, IDSKE ROEST

TOO MUCH INFORMATION? THE USEFULNESS OF TALKING REDUNDANTLY

THOMAS VAN DER LEER | LINGUISTICS

Suppose you and your favourite MoL-student are procrastinating on your homework assignment (very unrealistic, I know). You decide to play a game – not a very hard one, because your assignment is already difficult enough: one of your laptop screens shows a display containing multiple objects, each of a different shape and colour.

Your task is to think of one of the objects and have your teammate click on it. You are allowed to talk, but you can't point to the shape or give any other kind of non-verbal hint. Suppose you choose the star in the upper left corner. What do you say to your teammate? Think about it for a minute, before you continue reading.



(Rubio-Fernández 2019, 5)

You probably came up with a number of possibilities for referring to the star. For instance, you could have said 'the star', 'the upper left shape', 'the blue shape', 'the shape left of the rectangle', and so on. Clearly, though, these different descriptions are not all equally suitable for the task at hand. You probably went for a very short one, ensuring your teammate selects the right object without overcomplicating things.

Perhaps, you chose 'star'. If so, then Grice would be very happy for you. According to Grice (1975), rational and cooperative discourse participants normally choose their utterances in a way so as to be truthful (Maxim of Quality) and to be sufficiently informative while remaining as succinct as possible (Maxim of Quantity). Your description 'star' is as short as possible and refers uniquely to the object you had in mind. So, congratulations: according to Grice, you have just shown evidence of being rational and cooperative!

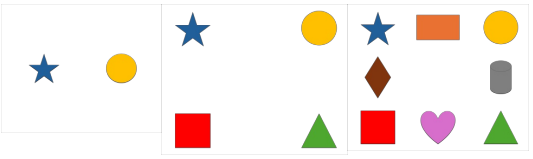
Hopefully then, you did not decide to say something like 'blue star'. Although such an utterance satisfies the Maxim of Quality, it clearly violates the Maxim of Quantity. Why say 'blue star' if there is only one star? You could have saved yourself the trouble by just not mentioning the colour at all, in which case your teammate would still have understood which object you had in mind. The Gricean model assumes that if one is cooperative, one adheres to the maxims. You didn't – ergo, you were being uncooperative.

That's probably not very nice to hear. Luckily for you, however, you're far from alone. In a series of experiments, Rubio-Fernández (2016, 2019) let participants play precisely the game described above. Interestingly, rather than choosing to say 'star', most participants decided to go for 'blue star'. Like you may have done, they violated the

Maxim of Quantity. This would render them uncooperative in the Gricean model – but what does that mean?

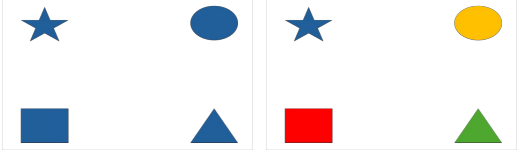
If someone is uncooperative, we'd expect them to be so because they are lazy or unmotivated and won't really bother to make clear what exact object they had in mind. However, here we call people uncooperative because they do the exact opposite: they give too much information. Why would we ever be uncooperative in this very unintuitive way? Clearly, there has to be a reason for our redundant behaviour – one that upholds our rationality as discourse participants.

To understand why we decide to be redundant, we should first ask ourselves in what kind of contexts we actually make this decision. In the same series of experiments, Rubio-Fernández (2019) found that increasing the number of objects on display caused the participants to mention the colour next to the shape of the target object more often. So, they were increasingly more likely to say 'blue star' (instead of just 'star') in displays with two, four and eight differently coloured shapes:



(Rubio-Fernández 2019, 5)

She additionally found that changing the colour of the other shapes to that of the target object decreases the chance someone still mentions the object's colour. Thus, people are much less likely to say 'blue star' in the left display than in the right:



(Rubio-Fernández 2019, 5)

Degen et al. (2020) moreover discovered that it also matters how typical an object's feature is for it to be mentioned redundantly. For instance, you probably wouldn't mention a banana's colour when that banana is yellow, whereas maybe you would when it's, say, purple (see also Westerbeek, Koolen, and Maes 2015). Furthermore, not all kinds of features are equally likely to be mentioned redundantly. For instance, we much more often talk about an object's colour than about the material it's made of, or about its size.

Thus, in choosing to be redundant, we seem to consider several factors. We can group these factors into two broad categories. Factors having to do with the visual salience of the object, such as the observed effect of adding more objects to the display or changing their colours, can be termed bottom-up: they only arise given empirical data in a specific context. In contrast, we can say that other factors such as typicality and the kind of the object's feature are top-down, since they relate to our world knowledge and previous experiences about objects and their features (Mitchell, Reiter, and Van Deemter 2013).

With our distinction between bottom-up and top-down factors in hand, let's return to the question we raised before: why do we choose to be redundant? There are two popular answers to this question. The first could be called the Continuous Semantics Account or CSA for short (Degen et al. 2020). The CSA departs from a continuous view on semantics: all utterances are 'noisy' to a certain degree. An utterance is said to be noisy when it's not immediately clear what is meant by it, e.g. because it has more than one meaning or because it's an unusual way of referring to that object. CSA sees being redundant as a way of potentially reducing this noise. Thus, giving more

information than strictly necessary can help take away some of the confusion on what you intended to convey.

Another explanation is given by the Visual Efficiency Hypothesis or VEH (Rubio-Fernández 2020). The VEH says that being redundant helps the listener find the target object more quickly and efficiently. Being redundant doesn't provide the listener with more information (contra the CSA), but it can still be very helpful for the listener's visual search. Multiple empirical studies support this idea, showing that listeners respond faster to redundant utterances compared to minimally informative ones (Arts et al. 2011, Paraboni, Deemter, and Masthoff 2007, Rubio-Fernández 2020).

I don't want to express a preference for either the CSA or VEH here. In fact, I believe we need both to fully understand our reasons for being redundant. It is no coincidence that the researchers interested in the top-down factors are proponents of the CSA, and those investigating bottom-up factors of the VEH. In a conversation I had with Robert Hawkins (one of the co-authors in Degen et al. (2020)), he explained to me how their notion of 'semantic noise' can perhaps best be seen as caused by a clash between the context and our world knowledge or expectations. Bananas are normally not purple, so just saying 'banana' to refer to a purple banana could lead to confusion. At the same time, the VEH can accurately account for the bottom-up factors we discussed: if more objects are added to a display, more time can be gained by redundantly mentioning an object's colour and with it quickly eliminating all objects not matching that colour.

To close off, let's zoom out and talk about more than shapes and colours. Redundant expressions cannot only be found in the context of reference games or purple bananas. Language is full of redundancy: we constantly tell each other stories that could have been told a lot quicker and give descriptions that could have been a lot more concise. What we have just learned, I think, is that the reason for this may be very allocentric: being redundant helps the people listening to us understand us more easily and quickly. So, the next time someone accuses you of TMI'ing, just know that you are a very rational, cooperative, and maybe even helpful human being.

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THE STORY OF MOSCOW'S METHODOLOGICAL CIRCLE

MATTEO MERCURI | PHILOSOPHY

The Moscow Methodological Circle was a philosophical, cultural, and political phenomenon which took part in the rebirth of the study of logic in the USSR, vastly influenced Soviet economy, and still has its place in today's Russian politics. It originated from the Moscow Logical Circle, which was created in the late 1950s by Aleksandr Zinov'ev, Boris Grušin, Merab Mamardashvili, and Georgij Ščedrovickij, at the Faculty of Philosophy of the University of Moscow. The leader of the group was Zinov'ev, a PhD student at that time, who became a prominent researcher in many-valued logic (see, e.g. (Zinov'ev ,1963)).

The study of logic in the USSR had been made possible again in 1946 - after being *de facto* banned since 1917 because outside of party and class interests. Stalin himself gave to logic studies gave a push with his articles on linguistics (Stalin 1950) in *Pravda*, the daily newspaper of the Communist Party of the Soviet Union (CPSU). Stalin had already showed an interest in logic in 1942, when he secretly asked philosopher Valentin Asmus to train a group of Red Army officers in formal logic (Roccucci 2024). Naturally, as soon as mathematical logic entered the scene, a major philosophical problem emerged in Soviet academia: its reconciliation with the dialectical logic that Marxism had inherited from German idealism. For this reason, the original aim of the Circle was to study the dialectical logic of Karl Marx's *Das Kapital*. But soon logic gained an entire new role: it became the sanctuary for those professors and students of philosophy who were looking for a research field free from the official government-imposed ideology (Roccucci, 2024).

Nevertheless, the original Circle did not last long: a contrast between Zinov'ev - who preferred to keep studying logic formally - and Ščedrovickij - who wanted to develop logic as a theory of thought in process - led to its collapse in 1954. Ščedrovickij later re-created the group as the Moscow Methodological Circle, now with a focus on the connection between logic and psychology, departing the group's attention also from dialectical materialism, and moving it to non-marxist philosophers, such as Cassirer, Husserl, and Popper (Roccucci, 2024). This was possible due to the new wave of (quite moderate) academic freedom that sparked after Nikita Chruščëv became the Secretary of the CPSU in 1953. Despite some problematic episodes, such as the censorship of Boris Pasternak's *Doctor Zhivago*, the Chruščëvian era was in fact one of generalised cultural rebirth in the Soviet Union. This period lasted until Chruščëv was ousted from power in 1964 by Leoníd Bréžnev, who brought back dogmatism and intolerance, repressing the cultural movements that had emerged in the previous years. This had direct repercussions also on the members of the Circle: Ščedrovickij was expelled from the CPSU in 1968 for speaking in favor of the authors of a *samizdat'*, that is, an unofficial publication of censored material that got hand-passed from reader to reader (see (Johnston,

1999)).

Despite this, Georgij Petrovič Ščedrovickij surely did not have the profile of the typical dissident, and was instead to become later an insider in Soviet power circles, well known for his mysterious and charismatic personality, that led one of his students to compare him to *Master and Margarita*'s Woland (Rindzevičiūtė 2015). Aside from being the greatest love story ever told, Michail Bulgàkov's *Master and Margarita* is *the* book on intellectual dissidence in the USSR. It was written from 1928 to 1940, during the Stalin era, and circulated as a samizdat' until 1966. The story goes like this: the devil, Woland, arrives in Moscow, ensuing chaos among Soviet society. The Master, a writer, has created a manuscript about Pontius Pilate, which leads to his persecution and eventual retreat into a mental institution. Before this, the Master and Margarita had lived together in happiness, deeply in love. However, the Master's troubles begin when his manuscript is rejected, leading him to burn the manuscript in despair. Margarita, devastated by the Master's disappearance, longs to be reunited with him. When Woland, the devil, arrives in Moscow with his retinue, chaos ensues. Margarita encounters Woland and his entourage, including the sinister cat Behemoth and the enigmatic Azazello. Woland offers Margarita a chance to find the Master if she agrees to serve as the queen of a surreal ball hosted by him, where she meets various damned souls, including Frida, a woman tormented by guilt for having killed her child. Frida's anguish is palpable, and Margarita feels deep compassion for her. Woland grants Margarita a wish. Instead of using it to reunite with the Master, Margarita selflessly chooses to save Frida from her eternal torment. Woland, impressed by Margarita's compassion and selflessness, ultimately helps her reunite with the Master anyway. But the Master cannot live without his manuscript, as this loss consumes him. Understanding this, Woland claims: "manuscripts don't burn", forging the greatest line in Soviet dissident literature, and restores the work that had been thought lost forever, so that the Master and Margarita are finally able to live together in peace. Just to say to what kind of character students compared Ščedrovickij: devilish, charismatic, decisively effective - the representation of the arbitrariness and opacity of power. Going back to our story, we should say that the oppression of the Bréžnevian era hit also Zinov'ev, the noble father of the moscovian Circle: he was exiled from the USSR in 1978 after the publication of his *Yawning Heights*, a satiric novel on Soviet society which brought him at the apex of dissident writers, second only to Aleksandr Solženicyn.

Despite this climate of repression, the members of the Circle managed to find an intellectual and practical space for developing their ideas in the spectrum between dissidence and orthodoxy. This approach characterised the methodologists in the period between 1964 and 1985. In these twenty years, Ščedrovickij led the group to develop a theory of thought as an embodied activity, always looked at as a part of a system. Accordingly, they developed a "methodology", intended as a means of formulating and coordinating group goals via formal scientific tools. Therefore, the methodologists started looking at decision theory and game theory, but they adopted an entirely different approach from the one developed in the West by institutions like the RAND Corporation. In fact, Ščedrovickij remained committed to the Marxian idea that scientific knowledge should be action-oriented (Rindzevičiūtė 2015). This approach brings him closer to western behavioral economics, and led to the development of a peculiar notion of

governance, based on an embodied and collective reason, that could be expanded via social and communicative activities. The Circle applied this approach to Soviet economy in collaboration with the government, becoming one of the most important management and consulting groups in Soviet planned economy. In particular, they started developing organisational activity games for the managers of state-owned corporations, factories, ministries, party bureaus, which approached Ščedrovickij and the methodologists to see their governance problems resolved. The games were seminars lasting several days, with the participation of groups ranging from fifty to two hundred people (Roccucci, 2024). Under the direction of the methodologists, the managers involved discussed some concrete problem, analyzing the situation, determining the objectives, and examining the dynamics that sparked among them, before proposing a solution. This way, Ščedrovickij became the most important consultant, or "management guru", as Rindzevičiūtė calls him, in the Soviet Union, conducting throughout the 1980s almost one hundred training sessions.

Ščedrovickij continued to develop and spread his ideas trough *perestroika*, the liberalisation of Soviet economy that started in 1985, and the fall of the USSR in 1991, bringing them into the new Russian market economy. After his death in 1995, his followers, above all his son Pëtr, were invited by Prime Minister Sergej Kirienko to apply the methodology to the new landscape of Russian politics, effectively making the the spin doctors of the coalition that brought Vladimir Putin to win the 1999 elections. During the electoral campaign, Pëtr Ščedrovickij led an organisational game designed to overcome the conflicts among the leaders of the coalition (Roccucci 2024), and in 2005 he was appointed director-general for strategy at the Russian nuclear authority Rosatom (Rindzevičiūtė 2015). To this day, he continues to train Russian managers, and to spread the methodologists' ideas in Russian society.

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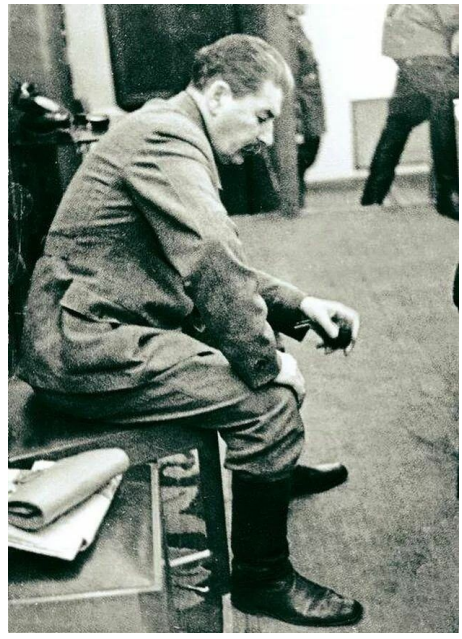


A CONCEPTUAL HISTORY OF BHK NEGATION

EDOARDO MENORELLO | PHILOSOPHY

INTRODUCTION

It is somewhat an item of common sense that sentences of natural language may at times be *false* rather than true. Yet, despite the obviousness of this observation, the role played by *falsity* in the context of modern logic is at best very modest. Indeed, the received view on the matter, dating back to Frege ((Frege 1919, 1979) "Logik"), defends that falsity is a psychological notion, but not a logical one, because all instances of false propositions can be circumvented by treating them as instances of true propositions prefixed with a negation operator. In this article, we argue that falsity holds his spot as a substantial logical notion in the context of intuitionistic logic. We do so by first by presenting the role of false proposition in Franz Brentano's reform of traditional logic, which stands in strong opposition to the received view. We then move to the BHK semantics for intuitionistic logic (christened after Brouwer, Heyting, and Kolmogorov). This, we argue, inherits some fundamental traits of Brentano's treatment of false propositions, and thus constitutes a witness for a system where the notion of falsity is conceptually prior to the one of negation.

FALSITY IN BRENTANO'S SYLLOGISTIC

Brentano was active in Vienna at the end of the 19th century, initially a scholar of Aristotle, he then devoted himself to the systematic investigation of the notion of *intentionality*, which he recovered from the scholastic tradition and made the cornerstone of his very own doctrine of *descriptive psychology*. According to Brentano, intentionality constitutes the fundamental feature of all acts of consciousness, to the extent that:

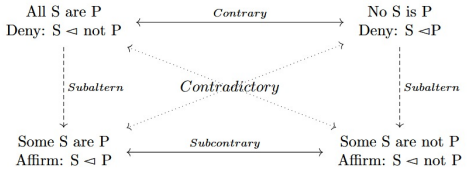
> *Every mental phenomenon is characterized by what the Scholastics of the Middle Ages called the intentional (or mental) in-existence of an object, and what we might call [...] reference to a content, direction toward an object. [...] In presentation something is presented, in judgement something is affirmed or denied, in love loved, in hate hated, in desire desired and so on. (Brentano 2012) .*

Accordingly, descriptive psychology aimed at providing a systematic description of the structure of all acts of consciousness, under the fundamental tenet that there could be no conscious act without a *correlative intentional object*, and all objects are such just because they could be correlative objects of some conscious act.

Brentano's interest in logic is a direct offspring of his philosophical ideas. Naturally, if all acts are determined by their correlative object, then the same should apply to the acts of judgments which were the specific domain of Aristotelian syllogistic. While traditional syllogistic held judgments to be of the form "S is P", Brentano held that the fundamental form of judgment is one of affirmation or denial of the existence of some representation, namely:

$$\varphi\ true = \text{Affirmation of } \varphi \& \varphi\ false = \text{Denial of } \varphi$$

In addition to those basic judgments of *thetic* form, Brentano also admitted *qualifying* judgments, that ascribed or denied some property of some object which has previously been affirmed. In (Brentano 1956), Brentano then showed how much of the traditional syllogistic was translatable into his systems, and how several of its shortcomings were there naturally amended. A simple and yet illustrative example, is Brentano's translation of Aristotle's square of opposition, which we report below in both versions, using ◃ for qualifying acts.



Following the cartesian tradition, Brentano defended that a representation φ could be affirmed whenever φ is *evident* to the subject, where *evidence* was taken as a primitive notion expressing the grasp of the undoubtable truth of the represented content.

What about φ false? Brentano held denial to stand for, *disappointment*[1] a primitive, undoubtable evidence of things not being in a certain way. However an act of denial is an act of consciousness, and, as such, it must have some correlative intentional object. If one now says that the correlative object of φ false is the situation that φ does not obtain, then they run into a contradiction, to the extent that φ false is in the end not an act of consciousness at all.

Brentano's solution was to say that to deny φ is to pose a certain modal qualification on acts representing φ. In particular, to deny φ is to say that any individual affirming φ, and that is, having evidence for the representation φ, must be doing so incorrectly.

Anyone who says, "No S is P" is thinking of someone judging that "An S is P," and declaring that in thinking of him in this way he is thinking of someone who judges incorrectly, someone who maintains something contradictory to his own judgement. (Brentano 2012).

Thus, an act of denial of some representation φ is grounded in the undoubtable awareness that things are not the way expressed by the content φ, but does not have φ or its negation as a correlative object. Instead, denial of φ is glossed as an affirmation about *any other possible individual*, stating that, insofar as they judge φ with evidence, they must be incorrect in doing so. As far as other individuals and their judgments are existing objects, contradiction is avoided.

Brentano would then go on to generalize this approach to all representations of negative form, so that negation itself would be understood as a linguistic fiction, whose logical structure is to be glossed in term of denial, and thus, of incorrectness of other individuals asserting the same content.

Now let us see how the logician can simplify these operations [...] All he has to do is to create the fiction that there are negative objects, too.[...]The fact that such fictions are useful in logic has led many to believe that logic has non-things as well as things as its object and, accordingly, that the concept of its

1. Terminology comes from (Husserl 1975).

object is more general than the concept of a thing. This is, however, thoroughly incorrect. (Brentano 2012)

The topic of falsity can hardly serve as a proper presentation of Brentano's brilliant contributions to logic and philosophy, and thus, we shall only notice here how the Brentanian treatment of false propositions fundamentally differs from the received view.[2] In particular, the latter reduces all instances of falsity to instances of negation, and thus rejects denial as logically substantial speech act. On the contrary, Brentano's position on negation endorsed the following two claims:

1. Negation as a propositional operator is dependent upon the speech act of denial, and;
2. denial of a representation φ can be glossed as a modal qualification on judgments of φ: if someone judges φ affirmatively, they are in that incorrect.

While the machinery of modern classical logic was mostly modeled after the Fregean approach, some of Brentano's ideas indirectly found their way into systems of non-classical logic. Two influential names among the disciples of Brentano were K. Twardowski and E. Husserl. While the former played a major role in the birth of the polish school of logic in Lvov-Warsaw, Husserl's ideas also had some influence in the development of certain strand of modern logic. It is well known, for example, that Arend Heyting came to know of Husserl's "Logical investigations" through Oskar Becker, and that it played some role in his formalization of intuitionistic logic. We now inspect Heyting's own take on negation, and argue it falls much closer to the Brentanian approach than to the received view.

NEGATION IN BHK SEMANTICS

Intuitionistic mathematics was initiated by the work of L.E.J. Brouwer, after the idea that mathematical objects are primarily mental construction of the working mathematician, and thus that mathematics should admit no non-constructive method of proof. Brouwer's ideas remained mostly informal, but the first formalization of the brouwerian principles was given by his disciple A. Heyting, resulting in the system nowadays known as intuitionistic logic.

The so-called BHK semantics for intuitionistic logic specifies the meaning of logical constant in terms of their contribution to the construction of mathematical objects. Heyting, in particular, introduces the semantics as follows:

A mathematical proposition expresses a certain expectation [...] Perhaps the word intention', coined by the phenomenologists, expresses even better what is meant here. We also use the word 'proposition' for the intention which is linguistically expressed by the proposition [...] The affirmation of a proposition means the fulfillment of an intention. (Benacerraf and Putnam 1983) (A. Heyting, The intuitionistic foundation of mathematics)

Thus, truth of a mathematical proposition means availability of evidence (fulfillment) for it, which, in the case of mathematics simply means availability of a proof. Since to have evidence of a proposition is *to know* it holds, and thus to be able to affirm it, Heyting's explanation implies the validity of the following identities:

$$\varphi\ true = \text{daffirmation of } \varphi = \varphi \text{ is evident} = \varphi \text{ is provable}$$

2. For a more general view of Brentano's logic, see (Parsons 2004, Simons 1992, Simons 2004).

According to this principle, Heyting lays down the explanation of logical constants as follows:[3]

⊥: There is no proof of ⊥,

φ & ψ: A proof of φ and a proof of ψ,

φ ∨ ψ: A proof of φ or a proof of ψ,

φ ⊃ ψ: A method yielding a proof of ψ from a proof of φ.

Prima facie, the semantics is silent about both falsity and negation. However, since we are given that truth is the same as affirmability, which is the same as provability, we must also endorse the following characterization of falsity in intuitionistic terms.

$$\varphi \ false = \text{denial of } \varphi = \varphi \text{ is disappointed} = \varphi \text{ is unprovable}$$

On the other hand, Intuitionistic logic does not typically take negation as a primitive constant, rather, it must be explained in term of something else. In particular, we have

$$\neg\varphi \equiv \varphi \supset \bot$$

which means that ¬φ true is the same as φ ⊃ ⊥ true, but what does the latter mean? Given the tables above, φ ⊃ ⊥ means that we have an effective procedure that given φ true yields ⊥ true. Plainly, because φ true means φ is provable, φ ⊃ ⊥ yields from a proof of φ, a proof of ⊥. Notice that, by definition, there is no evidence of ⊥ (in fact, the meaning of the constant ⊥ is defined by stipulating that it cannot be proven) so we have just shown with φ ⊃ ⊥ that there can be no proof of φ.

If we recall that a proof of φ is nothing but what makes φ evident, then it is easy to see how close we have landed to the Brentanian explanation. In fact, we have explained ¬φ as a method that yields evidence of ⊥ given evidence of φ, and that, given the meaning of ⊥, is in itself evidence of the fact that there can be no evidence of φ, which is the same as saying that, if anyone affirms φ evidently, they must be incorrect in doing so. In the case of Brentano, this impossibility statement was justified by appealing to *disappointment*. Because it is plain to the speaker that things are in a certain way, it is immediately given with this that they could never not be the way in which they evidently are, and so those who judge otherwise must be mistaken. In the case of BHK, instead, the impossibility statement is justified by the grasp of the meaning stipulated for ⊥. It was decided that ⊥ is a proposition without proof, so anybody who judges in such a way that allows to obtain a proof of ⊥ must therefore be mistaken. Whether and how far these two **justifications** are related, is not something that can be decided here, but it is important to notice that, according to ⋆ above, ⊥ is a proposition which is stipulated unknowable, and thus *cannot be affirmed, but only denied*, precisely because it has, by stipulation, no admissible proofs. Therefore, the justification for the construction of negative expression rests, both in the case of Brentano and of BHK, onto their respective explanation of the notion of falsity and denial.

In this sense, BHK semantics vindicates Brentano's claim 2: the negation of φ in BHK semantics is a modal statement about any other judgment of φ, qualifying it to the extent that they can never be evident.

3. The table is adapted from (Troelstra and Van Dalen 2014).

WHAT ABOUT BRENTANO'S CLAIM 1?

From $\star$ and the explanation of ¬φ it is easy to see that denial and negation are intuitionistically interderivable, i.e, we have:

$$\varphi \; false \iff \neg\varphi \; true$$

Does this entail the notions are identical? We contend no. Indeed, it is clear from the explanation of ¬φ just above, that understanding the meaning of ¬φ depends on the understanding of there being one proposition, ⊥, which can't be true but only ever false. Therefore, the understanding of affirmation of negative proposition presupposes the understanding of falsity of that same proposition, that is, its denial. On the contrary, the explanation of ⊥ can never, on pain of circularity, make reference to affirmation of sentences of negative form. In this sense, we conclude, the notions of *falsity* and *denial* are conceptually prior to the notion of affirmative negative proposition. This vindicates Brentano's claim 1 in the context of BHK semantics, and suggests it as a witness to the fact that falsity and denial deserve their own place in the architecture of modern logic.

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PROVING EVERYTHING WHILE REVEALING NOTHING: AN INTRODUCTION TO ZERO-KNOWLEDGE PROOFS

MARIANA RIO COSTA | COMPUTATION

INTRODUCTION

It's 2 AM and you're locked out of your email account. The system asks you to prove your identity by entering your password. You type it in, hit enter, and hope the server isn't compromised. Even if the connection is encrypted, even if the password is hashed, you've just revealed your secret to another computer. If that server gets hacked tomorrow, or if a malicious employee decides to peek at the database, your credential is exposed. For decades, this seemed like an unavoidable tradeoff: to prove you know something, you must reveal it.

In 1985, three researchers - Shafi Goldwasser, Silvio Micali, and Charles Rackoff - published a paper that would fundamentally challenge this assumption. Their work introduced the concept of zero-knowledge proofs (ZKPs): protocols that allow you to convince someone that a statement is true without revealing anything about why it's true or how you know it. The idea was so counterintuitive that the paper kept being rejected by multiple major conferences for 3 years. It would later earn them the first ever Gödel Prize.

Imagine logging into that email account without ever transmitting your password, not even in encrypted form. You simply convince the server that you possess the correct password, and the server becomes certain you're legitimate, yet learns nothing about the password itself. This isn't science fiction. This is what zero-knowledge proofs make possible.

WHAT IS A PROOF, ANYWAY?

Before we can understand zero-knowledge proofs, we need to step back and ask a more fundamental question: what is a proof?

Throughout our education, we've written countless proofs—on exams, in problem sets, in essays. Informally, a proof is something uttered by someone (the "prover") aiming to convince someone else (the "verifier") of the veracity of some statement. To formalize this, we can imagine there's some set $L \subseteq \{0,1\}^*$ (called a *language* in complexity theory), collecting all objects satisfying some relevant property. The prover's goal is to convince the verifier that some string x belongs to L.

You've probably encountered the complexity class NP before. Intuitively, NP captures languages that admit short, deterministic, non-interactive, and efficiently-checkable proofs. Think of a Sudoku puzzle: verifying a solution takes seconds, but finding it might take hours. The solution itself serves as a witness that the puzzle is solvable.

But what happens if we relax these assumptions? What if we allow the prover and verifier to interact

back and forth, exchange multiple messages, and use randomness (and a small probability of error) on the verifier's side? This leads us to the notion of *interactive proofs*.

INTERACTIVE PROOFS

Fix a language L and some string $x \in \{0,1\}^*$. In an interactive proof system, a computationally-unbounded prover $\mathcal{P}(x)$ and a Probabilistic Polynomial Time (PPT) verifier $\mathcal{V}(x)$ exchange messages back and forth. The prover's goal is to convince $\mathcal{V}$ that $x \in L$. At the end of the interaction, $\mathcal{V}$ outputs a bit b: output b = 1 means $\mathcal{V}$ believes $x \in L$, while b = 0 means $\mathcal{V}$ remains unconvinced.
Of course, for this interaction to be interesting we need some basic properties:

Completeness: If x truly belongs to L, then an honest prover should be able to convince the verifier. Formally, for any $x \in L$, we require $\Pr[\text{output}_{\gamma}(\mathcal{P}(x), \mathcal{V}(x)) = 1] = 1$.
Soundness: If x does not belong to L, then no prover—even a computationally unbounded cheating prover $\mathcal{P}$—should be able to convince the verifier, except with small probability. Formally, for any $x \notin L$ and any prover $\mathcal{P}^*$, we require $\Pr[\text{output}_{\gamma}(\mathcal{P}^*(x), \mathcal{V}(x)) = 1] \leq 1/2$.

Why bound the soundness error by 1/2? Because we can always reduce this error exponentially by repeating the protocol multiple times with fresh randomness. After k repetitions, a cheating prover succeeds with probability at most 2^{-k}.
We denote by IP the class of languages that admit complete and sound interactive proofs. It turns out that IP is surprisingly powerful. In fact, IP = PSPACE, meaning interactive proofs can verify any computation that uses polynomial space, even if it takes exponential time.

THE ZERO-KNOWLEDGE PROPERTY

We still haven't defined what it means for a protocol to be "zero-knowledge." Intuitively, a protocol should be zero-knowledge if the verifier learns nothing beyond the validity of the statement itself. More precisely, when the statement is true, the interaction gives $\mathcal{V}$ nothing that they couldn't have computed on their own without ever talking to $\mathcal{P}$.

To formalize this, we use a *simulation paradigm*. The idea is elegant: if the verifier could have simulated the entire interaction by themselves, then the interaction couldn't have taught them anything new. Consider the complete record of all messages exchanged during the protocol, we call this the transcript. We capture the simulation paradigm by requiring the existence of an efficient *simulator* S that, given only the input x, can produce a transcript that is computationally indistinguishable from a real interaction between $\mathcal{P}$ and $\mathcal{V}$.

Crucially, the simulator doesn't know any witness proving that $x \in L$. For NP languages, for instance, the simulator has no access to a certificate or solution, it only knows the statement x itself. This means a zero-knowledge proof reveals no information about *why* $x \in L$, only *that* $x \in L$.

There's one more subtlety: a dishonest verifier might deviate from the protocol in an attempt to extract some information. Imagine that someone, instead of following the prescribed steps, asks unexpected questions or responds in unusual ways, hoping to trick the prover into revealing the secret witness. Our definition must account for this possibility.

Let $\text{view}_{\gamma}(\mathcal{P}(x), \mathcal{V}(x))$ denote the verifier's view in an

interactive protocol, that is, everything the verifier observes during the execution, including all messages received and all random coins flipped.

We define an interactive protocol between a prover $\mathscr{P}$ and a PPT verifier $\mathscr{V}$ for a language L to be *perfectly zero-knowledge* if for any PPT verifier $\mathscr{V}^*$ (even a malicious one deviating from the protocol), there exists a PPT simulator S such that for any $x \in L$, the random variables $S(x)$ and $\text{view}_{\mathscr{V}^*}(\mathscr{P}(x), \mathscr{V}^*(x))$ are identically distributed.

This definition captures our intuition: no matter how the verifier behaves, they learn nothing they couldn't simulate themselves.

THE POWER OF ZERO-KNOWLEDGE

It's easy to see that every language in P (the class of problems solvable in polynomial time) has a trivial zero-knowledge proof: the prover and verifier can both just compute the answer themselves, and no interaction is needed.

But what about NP? Do all NP languages have zero-knowledge proofs? This seems much harder. After all, for an NP language, the verifier cannot efficiently determine membership on their own, they need a witness. How could we possibly prove membership without revealing information about that witness?

Remarkably, the answer is yes (assuming one-way functions exist). In a seminal result, Goldreich, Micali, and Wigderson proved that every language in NP has a zero-knowledge interactive proof. This was a stunning discovery: problems that seem to fundamentally require revealing a solution can actually be proven without revealing anything at all.
There is one catch: we must relax our notion of zero-knowledge slightly. Perfect zero-knowledge (where the simulated and real distributions are identical) is too strong. In fact, if all NP languages have perfect zero-knowledge proofs, then the polynomial hierarchy collapses, a consequence considered highly unlikely by most complexity theorists. Instead, the GMW protocol achieves *computational* zero-knowledge, where the simulated and real distributions are indistinguishable to any efficient algorithm.

AN EXAMPLE: GRAPH ISOMORPHISM

Let's see zero-knowledge in action with a concrete example: the graph isomorphism problem. Two graphs G_0 and G_1 on n vertices are *isomorphic* if there exists a permutation $\pi : [n] \to [n]$ such that applying π to the vertices of G_0 produces exactly G_1. The graph isomorphism problem asks: given G0 and G_1, are they isomorphic?

Here's a zero-knowledge protocol for proving that two graphs are isomorphic. Let $\pi^1 : [n] \to [n]$ be the isomorphism satisfying $G_0 = \pi^1(G_1)$, and let π_0 denote the identity permutation.

1. The prover $\mathscr{P}$ samples a uniformly random permutation $\pi^* : [n] \to [n]$ and sends $G = \pi^*(G_0)$ to the verifier $\mathscr{V}$.
2. The verifier $\mathscr{V}$ samples a random bit $b \in \{0, 1\}$ and sends b to $\mathscr{P}$.
3. The prover $\mathscr{P}$ responds with $\sigma = \pi^* \circ \pi^b$ (the composition of π^* and π^b).
4. The verifier $\mathscr{V}$ accepts (outputs 1) if and only if $\sigma(G_b) = G$.

In words: the prover sends a random permutation of G_0 to the verifier. The verifier then randomly challenges the prover to either (a) show how to get from G_0 to G (if b = 0), or (b) show how to get from G_1 to G (if b = 1). If the graphs are truly isomorphic, the prover can answer either challenge. If they're

not isomorphic, the prover can answer at most one of the two challenges.

You might wonder: why does the verifier need to make this random choice? Why not always send b = 1 (asking for a permutation from G_1 to G)? The reason is soundness against *dishonest provers*. If $\mathcal{V}$ always sent b = 1, a cheating prover could simply send a random permutation of G_1 as G in step 1, and they'd always pass the test even if G_0 and G_1 aren't isomorphic. The randomness prevents this attack.

THE NIKHEF PROTOCOL

Zero-knowledge proofs have revolutionized modern cryptography, enabling private authentication, anonymous credentials, and verifiable computation. But perhaps their most valuable application remains tragically theoretical: proving to your friends you're actually being productive in the MoL room without revealing the number of trips to Nikhef it took to get there.

The protocol would work as follows: you convince your friends you're being productive (completeness), while revealing nothing about your coffee consumption (zero-knowledge). Your friends are entirely satisfied with your work ethic, yet learn nothing about your Nikhef visit count. Unfortunately, the protocol breaks down when they ask about your progress and your response time is polynomial in the number of espressos required to process the question.

There's also a soundness problem: if you haven't actually been to Nikhef, no amount of cryptographic cleverness can simulate genuine productivity. A dishonest prover attempting this protocol will be caught with probability approaching 1 as the day progresses. The simulator, of course, can generate transcripts of perfect productivity without any coffee at all, but the simulator has the ability to rewind time, which would be far more useful for getting extra sleep than for faking work.

In conclusion: zero-knowledge proofs can hide what you know, but they can't create knowledge you don't have. And they definitely can't replace coffee. For that, you'll still need to make the trip to Nikhef :)

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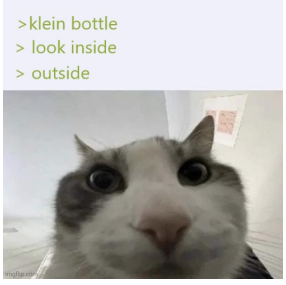
ROTATING SHAPES: TWO MÖBIUS MAKE A KLEIN

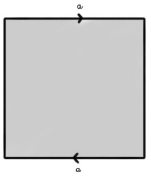
MAX WEHMEIER | MATHEMATICS

Most people know that if you take a long piece of paper, twist one end 180 degrees and then glue it to the other one, you get a shape with only one side and edge. Indeed, if you have never seen this, try it out right know! You can place your finger on one side or edge and follow it around until you end up on the “other” one. We call this shape a *Möbius strip* and would like to study it mathematically. To do so, we must abstract away the physical paper. From this, we will construct a Klein bottle, for which the “inside” and “outside” are also the same.



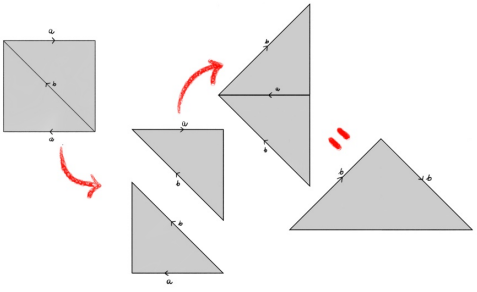
SURFACES

Sometimes drawing pictures is a valid method for constructing spaces, like the method of surfaces in topology. To abstract away the physical limitations of real paper, we just draw the piece of paper as a polygon and indicate which sides should be glued together. Edges with the same letter are identified in such a way that the arrows match. So this is a Möbius strip:

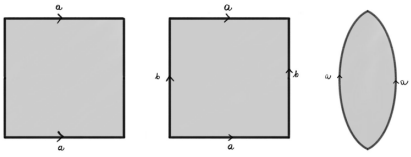


We start with a piece of paper, take two opposite edges and glue them together with a rotation of 180 degrees. We assume that this abstract paper is infinitely stretchy and sometimes even allowed to pass through itself. This is because in topology, we consider spaces only up to deformation. The latter is mainly needed in three dimension because some surfaces cannot exist otherwise.

Since we said that edges with the same labels should be glued together, we can also introduce new cuts and even split the piece into multiple as long as we keep track of how we cut. Similarly, we can identify the edges with the same label. So the following is still a Möbius strip and we see that it has only one edge, namely the one at the bottom:



We can also construct other interesting spaces this way. For example, if we do not rotate one side of our paper, we get a cylinder (left). If we also glue the other two edges together in a certain way, we get a donut (middle). We can also relax what a polygon is and see that the shape on the right is a sphere.

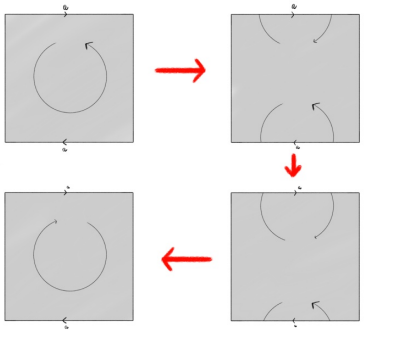


One nice thing about surfaces is that you can always recreate them using real paper (although the torus and sphere might pose some difficulties). So if you do not believe me, you can get a piece of paper and glue it accordingly to check.

A Möbius strip appears every time one wants to consider unordered pairs on the cylinder. We can parametrize the cylinder above by the unit square and divide by the equivalence relation generated by $(a, b) \sim (b, a)$. This corresponds to "folding" the representation of the cylinder along the main diagonal. Doing this results in the triangle representation of the Möbius strip we saw before.

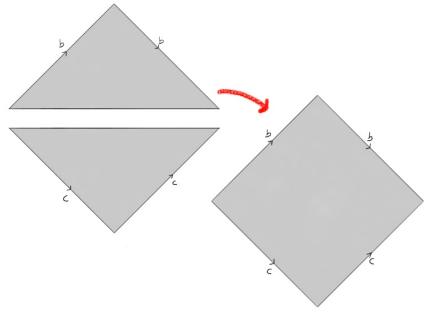
NON-ORIENTABILITY

In its real-life version, we have seen that a Möbius strip has only one side. Since we cannot define an "inside" or "outside", Topologists call this shape *non-orientable*. (Note that for example this does not hold for a cylinder.) We can already see this using the tools we have so far. Suppose we had a little loop with orientation living on the Möbius strip. Just by moving around, it can reverse its direction: This is a direct consequence of the twist in the paper. This cannot happen if we can define a clear inside or outside on the shape. In that case the loop can only stay on that side and moving around would never change its orientation since it is essentially moving on something that looks like $\mathbb{R}^2$. You can try with a cylinder to see this for example.

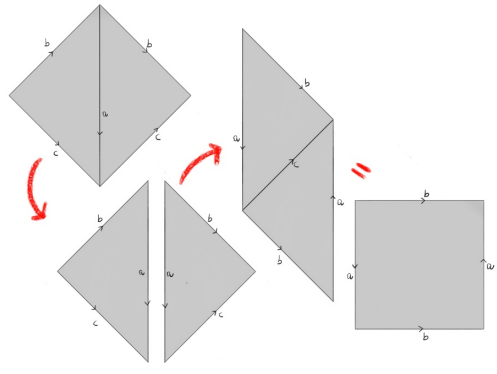


GLUING TWO MÖBIUS STRIPS TOGETHER

As we have already seen, a Möbius strip has only one edge. Therefore, there is a canonical way of gluing two Möbius strips together, namely along this edge:



You might notice that we could have also flipped the bottom one before gluing them together. You can easily check that we can use a similar construction get the same result. At first, this does not look like much, but like before, we can modify it to get:



This is the standard representation of a Klein bottle! We can again glue the edges with label b together to get a cylinder. Now we would like to glue the other two edges. But we cannot do it the same way as we did with the torus, because then the orientations do not match. Instead we need to somehow reverse the orientation of one of them. We can do this by pushing one end of the cylinder through one of the walls and pulling it through the inside to match the orientation. If you are unfamiliar with this, an animation might help: https://www.youtube.com/watch?v=E8rifKlq5hc. So we have shown that gluing two Möbius strips along their only edges gives indeed a Klein bottle! Using the same trick as with the Möbius strip, we can also see that a Klein bottle is non-orientable, explaining the meme at the beginning.

THE LONG JOURNEY TO CRAIG INTERPOLATION for PROPOSITIONAL DYNAMIC LOGIC

a story by Malvin Gattinger

this dynamic adventure starts in 1957

WHAT IS INTERPOLATION ?

To understand interpolation, we look at a natural language example:

IF GOD EXISTS, THE WORLD WILL NEVER END AND ALL HUMANS AND CATS WILL LIVE FOREVER

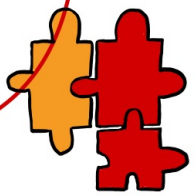
MIA IS A CAT

When we reason, we look for relevant pieces of information...

IF GOD EXISTS,

MIA WILL LIVE FOREVER

...to form our conclusions.



In logic, the relevant pieces come together in the interpolant

$$\left.\begin{array}{l}\phi \to \theta \\ \theta \to \psi\end{array}\right\} \phi \to \psi$$

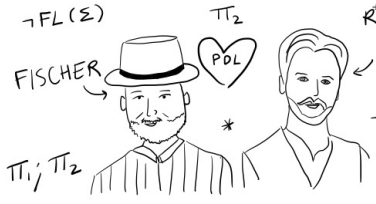
$$var(\theta) \subseteq var(\phi) \cap var(\psi)$$

IF WE CAN FIND SUCH A θ FOR EVERY $\phi \to \psi$, THE LOGIC HAS CRAIG INTERPOLATION

SOME LOGICS HAVE CRAIG INTERPOLATION, SOME DO NOT...

CRAIG APPROVED LOGICS:
- FOL
- CPL
- IPL

In 1979, two men began to see stars...



... and Propositional Dynamic Logic (PDL) was born.

CARTOON: JOSJE VAN DER LAAN

NEW LOGICS = NEW QUESTIONS

But the most important...

DOES CRAIG APPROVE?

Over the years, many have tried to prove interpolation for PDL.

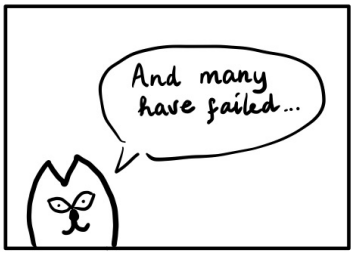
1979 ...  ... 2004

HALL OF FAME

Leivant, 1981

Borzechowski, 1988

Kowalski, 2004



* 10 years later... *

To be continued...

CARTOON: JOSJE VAN DER LAAN

ALSO WANT TO GET CREATIVE FOR THE ILLOGICIAN?
LET US KNOW!

SEND AN EMAIL TO JOSJE.VAN.DER.LAAN@STUDENT.UVA.NL

THE AXIOM OF BAD CHOICE AND ITS EQUIVALENTS

MATTEO CELLI | MATHEMATICS

TRANSLATOR'S NOTE

This paper, unpublished on any known mathematical outlet at the time, dates back to March 1946, and was found in a parcel in the Beth Library, in the ILLC common room, in the Autumn of 2025. Most plausibly, the author unsuccessfully attempted publication multiple times, culminating in a request to E.W. Beth himself. We publish it here, for the first time in almost eighty years, as we believe it is very well worth the attention of the open-minded logicians of the twenty-first century. Below we print what we hope to be the most faithful rendering possible of the original German. Much gratitude is owed to Max Imilian Wehmeier for his help in the translation process.

FOREWORD

My Dearest Evereth,

I hope this letter finds you well. I am sending you what I believe to be a legitimate, and indeed nothing short of groundbreaking, alternative foundation of Mathematics, namely Mennerl Set Theory with the Axiom of Bad Choice (MBC). You are my last hope for this work to be published: thus far, any ambition in this regard has been thwarted by the viciousness of Zermelo and his followers, an avid sect who keeps sabotaging my endeavours in order to retain cultural hegemony in mathematical foundations.

I believe your renowned wisdom will leave no room for doubt that this short notice of mine has the potential of shaking Mathematics from its foundations for the years to come. Indeed, I claim that MBC constitutes a more natural foundation for Mathematics than its more optimistic counterpart ZFC, since if Mathematics is to be an adequate modelling tool for the empirical world, it cannot but be able to account for the existence of universal bad luck, and the utmost chaos of the states of affairs human beings find themselves involved in.

I hope to hear your opinions soon,

Sincerely yours

E. Mennerl, Göttingen 1946

THE AXIOM OF BAD CHOICE AND ITS EQUIVALENTS

In the present paper we outline a novel axiomatisation of Set Theory, *Mennerl Set Theory with Bad Choice* (MBC), which, as you may guess, is named after the author, who rightfully deserves recognition for such a contribution. Intuitively, the Axiom of Bad Choice states that for any collection of non-empty disjoint sets, there exists a function that picks exactly those elements that we do not want, and sends them to a thoroughly useless set we know nothing about.

We present a new suitable axiomatisation of Set Theory that includes the Axiom of Bad Choice (henceforth ABC), and prove its equivalence with the *Disordering Principle*. We conclude by outlining its most widely applicable corollary, namely the *Murphy Law*.

For this purpose, I add a "desirability predicate" D to the language signature of $\mathscr{L}_{\in}$. This will allow the

expanded language $\mathscr{L}_D$ to distinguish whether we care about a certain set, or better, if the existence of a certain set is in any way beneficial for our purposes. Notice how powerful and natural the addition of such predicate is: why would one not want to state whether a given set is desirable? Only one who has a political interest in preserving the depreccable Zermelian *status quo*. Given that in MBC we hold that everything in our theory is a set (that is, we do not accept *urelemente*), by adopting it as foundational paradigm we will be able to state wheher or not the existence of *any* mathematical object is desirable. I believe that, out of respect for the reader's intelligence, the advantages yielded by such expansion of the language need no further explanation.

Indeed, we are now equipped to formulate the *Axiom of Bad Choice*:

$$\forall X(\forall y(y \in X) \rightarrow y \neq \emptyset) \rightarrow$$
$$\exists f(\neg Df \wedge f : X \rightarrow \bigcup X \text{ s.t. } \forall y \in X(\neg Dy \wedge f(y) \in y))$$

The desirability predicate D allows us to express the intuitive idea I hinted at at the beginning of the paper: for any collection of non-empty disjoint sets, there exists a non-desirable function from X to its union $\bigcup$X, which selects a non-desirable element from any set, thus implying the existence of a non-desirable set. We call this function a *disruptor* and its range *Jinx*. For the reader whose senses are offuscated by their unwillingness to taking me seriously, I should like to remind them that no other than Kurt Gödel came up with a similar principle, which he called "universal bad luck". Here is a quote, from *Ueber die Ausmaße des Unglücks in der Mengenlehre nach Zermelo-Fraenkel und verwandter Systeme* (Gödel (1931)):

"Bad luck is pervasive in all mathematising, and it would be foolish for any mathematical project not to acknowledge its existence. A day will come in which this stance, far from controversial, will be nothing short of a truism for the working mathematician"

If you do not believe me, I hope you will at least believe Gödel. Who, if not him, can speak of bad luck, after having discovered that the project of finding a way to prove all arithmetical truths is doomed by incompleteness? This is my appeal: if you accept incompleteness, you should also wholeartedly endorse bad luck. If you are honest enough to do so, its first ever formalisation and embedding into the beating heart of mathematics will come to you as a breath of fresh air. Furthermore, I will show it is equivalent to an uncontroversial principle, namely the *Disordering Principle*. Let us begin by rehearsing some preliminary definitions:

Definition [Chaotification]

Let any R ⊆ W × W for a given domain W. Then, let C(R) be the least subset S of W × W containing R, such that ¬DS. We call this the chaotification of R.

Definition [Disordering of an Ordered Set]

Let (P, R) be any ordered set. Then, (P, C(R)) is the disordering of (P, R)

Which allow us to finally define the following:

Disordering Principle

Any well-ordered set can be disordered.

Theorem.

$$ABC \iff \text{Disordering Principle}$$

Proof. The "if" direction is trivial - indeed, I claim that not immedialtely seeing why this is the case denotes a lack of understanding such that no objections can be taken seriously. Hence, the proof of this entailment is left as an exercise to the reader. We are therefore only left to show the "⇐" direction. For this, it is crucial to notice that the

disruptor can be seen as the *chaotification* of an arbitrary f. Therefore, consider X a collection of non-empty disjoint sets. Then, by *Pairing, Union, Replacement* and *Powerset*, we know $X \times X$ is a set. Therefore, for any $f \subseteq X \times X$, we can define its *chaotification* C(f), thus obtaining the desired *disruptor* function. □

In MBC we can easily prove the *Murphy Conjecture*, first proposed in 1799 by the enlightened British matemathician Lord Percival Murphy in his seminal (Murphy, 1799), which now fully deserves its upgrade to the name of *Murphy Law*. Murphy stated that "for any mathematical structure with a given property, there must be one that has utterly monstrous and undesirable features. If this could be proven, mathematics will finally be able to be a universal language not only for Science, but for any endeavour of humankind."

Now, take any set of mathematical structures, and let φ define a subset S defined by replacement on such set. Now consider S* a partitioning of S such that all its elements are disjoint. By Bad Choice, we obtain a *jinx* set of undesirable elements, i.e., the set of Murphy's "utterly monstrous and undesirable" structures, constituting a chaotification of the property φ.

Having proven the Murphy Law, as per Lord Murphy's prophecy, nothing can stop Mathematics from providing a universal foundation of human endeavours. The last objection I will address is that of the Brouwerians, who will reject my axiom on grounds of its non-constructive nature. Fools! To answer their scepticism I will only highlight the ridicolous optimism of constructive mathematics: do they *really* believe that mathematics can be fully directioned by the human mind? They speak as if no calamity ever happened to them. To someone who is so blind to refuse to acknowledge the sovreignity of bad luck, I cannot but spitefully turn my back against. For the few enlightened individuals willing to recognise the uncontrovertibility of my results, on the other hand, wisdom and liberation awaits.

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ILLUSTRATION: HAOYU WANG

THE PINEAPPLE INCIDENT: A BELIEF-THEORETIC AND TOPOLOGICAL ANALYSIS

GIUSEPPE MANES | COMPUTATION

In the famous TV series *How I Met Your Mother*, Episode 10 of Season 1, the so-called *Pineapple Incident* presents an epistemologically rich mystery: protagonist Ted Mosby awakens with complete amnesia, a sprained ankle, an unknown woman in his bed, and a mysterious pineapple on his bedside table. This article employs the *Multi Layer Belief Model*, developed in (Pinto Prieto 2024), to formally analyze this narrative as a case study in belief formation under partial, uncertain, and contradictory evidence.

The model integrates qualitative and quantitative evidence processing, drawing on the topological framework for evidence structure and Dempster-Shafer theory for evidence combination. We will use, respectively, as sources of this framework (Özgün 2017) and (Shafer 1976). While the formal computation of topologies and belief functions is performed according to the models' specifications, this article presents only the resulting belief degrees for key hypotheses (the woman's identity, the pineapple's origin) rather than detailing the computational steps.

The results will illustrate how different epistemic attitudes yield rationally defensible yet distinct conclusions when faced with the same body of evidence.

MULTI-LAYER BELIEF MODEL

First, we present the framework in question. The foundation of our framework is a **qualitative evidence frame** $(S, \mathscr{E})$, where S represents the set of possible states of the world, and $\mathscr{E} \subseteq 2^S$ is a collection of evidence sets. Each element $E \in \mathscr{E}$ represents a basic piece of evidence—a proposition about which states are compatible with that particular evidential constraint.

From this evidential subbase $\mathscr{E}$, we generate a **topology** $\tau_{\mathscr{E}}$, defined as the collection of all arbitrary unions of finite intersections of elements from $\mathscr{E}$. Formally, we can firstly define the basis $\mathscr{B}_{\mathscr{E}}$ by considering closing the se of basic pieces of evidence under intersection:

$$\mathscr{B}_{\mathscr{E}} = \left\{ \bigcap E : E \subseteq 2^{\mathscr{E}}, |E| \in \mathbb{N} \right\}.$$

Now, we can construct the topology by closing $\mathscr{B}_{\mathscr{E}}$ under unions

$$\tau_{\mathscr{E}} = \left\{ \bigcup F : F \subseteq \mathscr{B}_{\mathscr{E}} \right\}$$

This topology represents the space of all possible *arguments* that can be constructed from the available evidence. Each open set in $\tau_{\mathscr{E}}$ corresponds to a proposition that can be justified by some combination of the basic evidence pieces.

The first layer introduces the concept of a **frame of justification** $\mathscr{J} \subseteq \tau_{\mathscr{E}} \setminus \{\emptyset\}$, which formalizes an agent's evidential demands. While $\tau_{\mathscr{E}}$ contains all

possible arguments, $\mathscr{J}$ specifies which of these arguments the agent actually accepts as valid justifications for belief.

Two fundamental frames illustrate the spectrum of evidential demands:

• **Dempster-Shafer frame** $\mathscr{J}^{DS} = \tau_{\mathscr{E}} \setminus \{\varnothing\}$ accepts all possible arguments

• **Strong Denseness frame** $\mathscr{J}^{SD}$ accepts only arguments consistent with all available evidence

A set T is a **justification** for proposition P with respect to $\mathscr{J}$ if and only if $T \subseteq P$ and $T \in \mathscr{J}$.

The quantitative layer handles uncertainty through a **quantitative evidence frame** $(S, \mathscr{E}^Q)$, where $\mathscr{E}^Q \subseteq \mathscr{E} \times [0,1]$ associates to each evidence set a degree of certainty. We define a **mass function** $\delta : 2^{\mathscr{E}} \to [0,1]$ that distributes certainty across evidence combinations:

$$\delta(\mathbf{E}) = \prod_{E \in \mathbf{E}} p(E) \prod_{E \notin \mathbf{E}} (1 - p(E))$$

This function ensures that the total mass sums to 1, with each piece of evidence's certainty distributed across all its possible co-occurrences with other evidence.

The bridging layer connects qualitative and quantitative aspects through **evidence allocation functions** $f : 2^{\mathscr{E}} \to \tau_{\mathscr{E}}$. These functions determine how sets of evidence are interpreted as arguments, subject to three rationality constraints:

1. Uncertainty preservation: $f(\varnothing) = S$

2. Coherence: f(E) must be in the topology generated by E and dense or empty

3. Uniformity: For any $E \subseteq \mathscr{E}$ and $f, g \in \mathfrak{F}$, either $f(E) \subseteq g(E)$ or $g(E) \subseteq f(E)$

Common allocation functions include:

• **Strict interpretation** (i): Maps to intersections of evidence

• **Moderate interpretation** (u): Maps to unions of evidence

• **Minimum dense interpretation** (d): Maps to minimal dense sets

The framework culminates in the **multi-layer belief function**:

$$Bel_{\mathscr{J}}(f, P) = \sum_{A \subseteq P} \delta_{\mathscr{J}}(f, A)$$

where $\delta_{\mathscr{J}}$ normalizes mass values relative to the chosen justification frame. This function computes degrees of belief that respect the agent's evidential demands ($\mathscr{J}$), interpretive stance (f), and handles uncertainty from potentially contradictory evidence.

The framework's power lies in its ability to model diverse epistemic attitudes while maintaining formal rigor, generalizing both Dempster-Shafer theory and topological evidence models as special cases.

Let us now employ this framework for the reconstruction of the story.

THE PINEAPPLE INCIDENT

The story began at MacLaren's Pub, where Ted Mosby consumed five "Red Dragon" shots under the enthusiastic urging of his friend Barney Stinson. This initial event established the first piece of formal evidence

E_1 = {Ted drank 5 Red Dragon shots}

with certainty $p_1 = 0.95$,

witnessed by four reliable observers.

Source 1 - Marshall and Lily

What followed was a sequence of events that Ted's friends would later reconstruct for him. Marshall

Eriksen and Lily Aldrin testified that Ted made three increasingly incoherent phone calls to Robin Scherbatsky, providing

E_2 = {Ted made 3 drunk calls to Robin}

with $p_2 = 0.9$,

supported by both phone records and witness confirmation.

During his intoxicated performance, Ted fell from a table, spraining his ankle

E_3 = {Ted fell off table, injuring ankle}

with $p_3 = 0.85$,

corroborated by physical evidence the following morning.

Marshall and Lily then brought Ted home at 1 am and put him to bed alone

E_4 = {Friends brought Ted home at 1 am}

with $p_4 = 0.9$ *and*

E_5 = {Ted was put to bed alone} *with* $p_5 = 0.85$,

both from consistent eyewitness accounts.

Certain facts

The morning presented physical evidence that would define the mystery. Upon waking, Ted discovered an unknown woman sleeping in his bed

E_6 = {Unknown woman in Ted's bed} *with* $p_6 = 1.0$,

an undeniable physical fact.

Beside him sat a perfectly ripe pineapple on his bedside table,

E_7 = {Pineapple on bedside table} *with* $p_7 = 1.0$,

equally undeniable.

Ted's suede jacket was burnt,

E_8 = {Burnt suede jacket} *with* $p_8 = 1.0$,

physically present.

His arm bore writing: "*Hi, I'm Ted. If lost, please call...*",

E_9 = {"Hi, I'm Ted..." written on arm} *with* $p_9 = 1.0$,

physically verified.

Source 2 - Carl

Testimonial evidence began filling the gaps. Bartender Carl revealed via telephone that before leaving the bar, Ted made a final call inviting someone over

E_{10} = {Ted made final call inviting someone over}

with $p_{10} = 0.9$,

from Carl's direct recollection.

Carl also mentioned that Ted had expressed wanting to sneak into the zoo to see penguins

E_{11} = {Ted mentioned wanting to see penguins at zoo} *with* $p_{11} = 0.7$,

a hazier memory.

Source 3 - Trudy

After a while the woman next to Ted in the bed wakes, up revealing that is not Robin, but a girl named Trudy. She explained that she and Ted met at the bar after her recent breakup

E_{12} = {Trudy met Ted at bar after breakup}

with $p_{12} = 0.95$,

a clear first-person account. They exchanged phone numbers in the ladies' room

E_{13} = {They exchanged numbers in ladies' room}

with $p_{13} = 0.95$,

similarly clear.

She received Ted's invitation call

E_{14} = {Trudy received Ted's invitation call}

with $p_{14} = 0.95$,

with phone record confirmation. And she came to Ted's apartment that night

E_{15} = {Trudy came to apartment} *with* $p_{15} = 0.9$,

from her direct testimony.

Source 4 - Robin

After that Robin Scherbatsky also called Ted back, confirming what he had already discovered. Indeed, she attended a charity dinner all evening

E_{16} = {Robin was at charity dinner all evening}

with $p_{16} = 1.0$,

with multiple confirming witnesses.

Source 5 - Barney

Finally, Barney Stinson admitted he set Ted's jacket on fire as punishment for calling Robin again

E_{17} = {Barney set jacket on fire} *with* $p_{17} = 0.85$,

from his confession. And that he then slept in Ted's bathtub

E_{18} = {Barney slept in bathtub} *with* $p_{18} = 0.9$,

physically verified that morning.

FORMAL COMPUTATION OF BELIEF

This collection of evidence forms the **qualitative evidence frame** $(S, \mathscr{E})$, where S represents all possible states concerning the events of that night, and $\mathscr{E} = \{E_1, ..., E_{18}\}$ constitutes the evidential base.

The corresponding **quantitative evidence frame** is $(S, \mathscr{E}^Q)$, where $\mathscr{E}^Q = \{(E_1, p_1), ..., (E_{18}, p_{18})\}$ associates each piece of evidence with its degree of certainty.

After collecting all the pieces of evidence and reconstructing the story, we can now retrospectively analyze the main hypotheses that arose during the evening. We will consider the possibilities that we encountered during the night: first, that the woman in Ted's bed might be Robin; second, the conclusion ultimately revealed, that the woman is actually Trudy; and finally, the hypothesis regarding the origin of the pineapple, for which only indirect or minimal evidence exists.

By reconstructing these hypotheses in light of the full narrative, we can assess whether the beliefs we formed at the moment matched the actual events, identify which hypotheses were fully justified, which were contradicted, and which remained indeterminate due to insufficient evidence.

Hypothesis H_1: "The woman is Trudy"

For a cautious evaluator, we include only the evidence that directly reflects the actual events: Trudy's testimony and the physical presence of the woman in Ted's bed. Formally, using the **intersection allocation function** i:

$$i(\{E_{12}, E_{14}, E_{15}, E_6\}) = E_{12} \cap E_{14} \cap E_{15} \cap E_6$$

This intersection is non-empty and fully consistent. Computing the belief with respect to $\mathscr{J}^{SD}$:

$$Bel_{\mathscr{J}^{SD}}(i, H_1) = \sum_{A \subseteq H_1} \delta_{\mathscr{J}^{SD}}(\{E_{12}, E_{14}, E_{15}, E_6\}, A) \approx 0.94$$

For the permissive evaluator, we use the **union allocation function** u to include weaker evidence, including Ted's belief that he was calling Robin (E_{10}):

$$u(\{E_{10}, E_{12}, E_{14}, E_{15}, E_6\}) = E_{10} \cup E_{12} \cup E_{14} \cup E_{15} \cup E_6$$

The corresponding belief under $\mathscr{J}^{DS}$ is:

$$Bel_{\mathscr{J}^{DS}}(u, H_1) \approx 0.96$$

Even when considering all information permissively, the evidence overwhelmingly supports that the woman in bed is Trudy. Ted's initial belief about calling Robin is treated as narrative context rather than contradictory evidence.

Hypothesis H_2: "The woman is Robin"

Here the evidence presents a direct contradiction. Robin was confirmed to be at a charity dinner all

evening (E_{16}), while a different woman was in Ted's bed (E_6). For the cautious evaluator:

$$i(\{E_2, E_6, E_{16}\}) = E_2 \cap E_6 \cap E_{16} = \varnothing$$

$$Bel_{\mathscr{J}^{SD}}(i, H_2) = 0$$

For the permissive evaluator, we consider the union of available evidence:

$$u(\{E_2, E_6, E_{16}\}) = E_2 \cup E_6 \cup E_{16}$$

$$Bel_{\mathscr{J}^{DS}}(u, H_2) \approx 0.25$$

While the permissive evaluator allows for minimal support due to weaker justifications, the cautious evaluator rules out this hypothesis entirely.

Hypothesis H_3: "Pineapple came from the zoo"

The relevant evidence here is minimal and entirely indirect:

$$i(\{E_7, E_{11}\}) = E_7 \cap E_{11} = \varnothing$$

No direct connection exists between the physical pineapple and Ted's expressed interest in penguins. Under the strict $\mathscr{J}^{SD}$ frame:

$$Bel_{\mathscr{J}^{SD}}(i, H_3) = 0$$

Under the permissive $\mathscr{J}^{DS}$ frame with union allocation:

$$u(\{E_7, E_{11}\}) = E_7 \cup E_{11}$$

$$Bel_{\mathscr{J}^{DS}}(u, H_3) \approx 0.18$$

CONCLUSION

The Multi Layer Belief framework produces a precise epistemic landscape of the Pineapple Incident:

Hypothesis	Cautious Agent ($\mathscr{J}^{SD}$)	Permissive Agent ($\mathscr{J}^{DS}$)
Woman=Trudy	0.94	0.96
Woman=Robin	0.00	0.25
Pineapple explained	0.00	0.18

The evidence strongly supports Trudy's identity across all epistemic attitudes due to **multiple consistent, independent testimonies** that form a coherent narrative. Ted's initial belief that he was calling Robin is now treated as narrative context, so it no longer introduces any conflict for a cautious agent. The Robin hypothesis collapses under the framework's contradiction detection mechanism. The pineapple's origin remains epistemically inaccessible—the model formally demonstrates why no degree of open-mindedness can overcome **fundamental evidential insufficiency**.

The pineapple's enduring mystery is thus not a failure of investigation or reasoning but a natural outcome when evidence is simply not available to support a specific, coherent belief. This reality is perfectly captured by the multilayer model's capacity to rigorously distinguish between conclusions that are justified by the weight and consistency of evidence and those that, however narratively appealing, lack the necessary evidentiary foundation.

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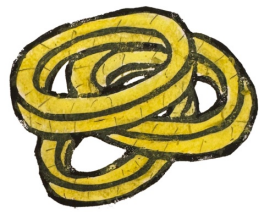
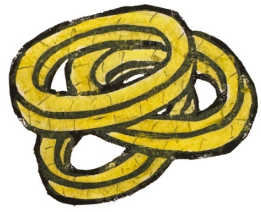




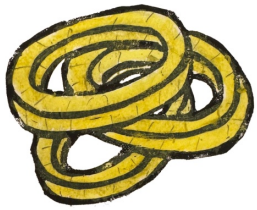


ILLUSTRATION: HUGO RENNINGS

LOGICAL RHETORIC MAKES ILLOGICAL DEBATES

ALEXANDRE MAZUIR | PHILOSOPHY

DEBATES AS LOGIC

When defining a logical system, amongst other things, we define axioms and inference rules. From the application of the latter to the first, we aim to prove validity of statements in that logical framework, the holy grail of that process, the true goal of that ultimate quest, being the sanctified tautology, a statement true in itself (although bound by the system, but not dependent on any other), a stable result, unquestionable. The parallel between this structure and that of an argument, taken in the sense of a discussion, could not be clearer: speakers argue for certain ideas (their axioms) to be considered, and, through applications of inference rules to that newly found common ground, both aim for the tautology, a statement the other cannot refuse, a conclusion going their way. And, of course, the meat, the heart, of the debate truly is the vivid game of "attacking" and "defending" one's axioms that ensues, done through evocation of counterexamples, meta-proofs of contradiction, so on and so forth. But why is it then that so many debaters focus on the inference rules instead?

Indeed, a great amount of discourse, notably coming from contrarians, seems not to focus on debating statements, assertions, but proving the opponent's inference to be wrong, ill-willed, and thereby not claiming anything of their own, no weight to bear themselves, simply attacking the assumed stability of the opposed discourse. I present here my (very surface-level) understanding of that phenomenon, argue for a looser, non-formal understanding of what a debate is, and tell you why I believe such a behaviour to be dangerous to discussions as a whole.

LOGIC OF DEBATES

First of all, let us consider that formal logic parable once again. I argue this focus on structure misses the point of real debate. In a previous essay (although written in godforsaken French), I considered a list of characteristics any interaction must obtain for them to earn that title. Most are irrelevant to our matter, but one of them is essential for us: debaters address a double discourse, directed towards both their opponent and the audience, attempting to convince the latter. Notice how the opponent can also themselves be part of the audience, making cases of debates behind closed doors still compatible with that criterion.

I have to say, however, that this definition is not ideal, but practical: it helps characterise what many debates end up nowadays going for. Listen to any political roundtable, for instance; for people represent parties and ideas they are not themselves allowed to concede to adverse arguments, and thus often playing deaf. The debate cannot be happening for them to change their mind. Crucially, this results in this double discourse being unbalanced, where debate between opponents is more akin to a

demonstration, a play, rather than an actual discussion, of which the real point is to convince the audience.

LOGIC IN DEBATES

But convincing the audience of what? Suppose there is an election, with two parties, the "Classicalists" and the "Intuitionist". Classicalists could either try to convince the audience of accepting its axioms or rejecting the ones Intuitionists argue for. But, there is another more insidious option: presenting the Intuitionist narrative, i.e. argumentation, as unreasonable, contradictory, illogical. And to do so, Classicalists would not aim for the throat of axioms Intuitionists defend anymore, but their goal would instead be to show that the inference rules Intuitionists apply to them are invalid. This would make Intuitionists appear unstable; and soon questions follow: yes, I may agree with their premisses, but how can I be sure they will derive the right conclusion from them? What good is there in someone defending what I believe if they do wrong with it?

And what I believe is that this argument is even stronger than advocating for axioms of your own. As mentioned before, it doesn't involve commitment to any ideas, it allows to present oneself as the great keeper of the well-order of the discussion, the repeller of all subtle fallacies and simplifications, an arbiter of sort, granting authority on the debate, even though one is still a participant. And all this can hardly be viewed in a negative light: serious debate should be kept to an upmost standard of validity, who would want to listen to politicians who don't even make sense? It is then completely possible, understandable, (and, here, sadly, the parable leaks heavily into reality) to have Classicalists uniquely attack Intuitionists' inferences, without presenting, nor defending, their own axioms, once again messing with the balance of the debate. An empty side of to the scale keeping an eye on it.

DEBATES FOR LOGIC

But much like in logic itself, where language and meta-language must crucially be different, axioms and inferences, one's premisses and argumentation, lie on different levels, they cannot blend. And focusing on inference rules is forgetting they only apply to the discourse itself, whereas the axioms apply to the concrete, in our case, to actual lives. In short, debating about the validity of an argumentation is really only debating about the debate, not what the debate is about.

One of the consequences of this meta-ification of debates is that they become far remote from people who do not typically engage in verbal jousts. Debates, and a fortiori most politics, get more and more estranged from those who care about the axioms, about life. This can be clearly observed in the upbringing of political figures, more often than not trained in communication and media, rather than spontaneously defending what they take at heart. Now, I perfectly understand one might prefer this "seriousness" for politics, though I personally do not agree, for the main goal of a traditional politician is representing their people, and this clear divide seems to contradict that first principle. Now, it might be a good time to remind ourselves that all of this was a specific case. We are still talking about debates in general. This political is but one of the ugly implications of how we treat discussions.

All that being said, what possible view of debates would help us avoid those issues? Well, a first approach would be to focus on what participants argue for. Notably, in our previous account, there is

nothing stopping us from considering a debate between two participants holding the same world view. As long as they argue and try to convince, it would still be a debate, and for they might never criticise the other's position (as it is their own) no contradiction would blatantly appear. I believe it is thus essential to keep in mind what participants defend (even implicitly) to grow healthy debates, and, as a plus, avoid unnecessary ones, the likes of what we just mentioned.

In the end, sometimes, the better definition is the simplest, the most intuitive one, even though it might be devoid of a bullet-proof logical structure, it still is what someone might initially believe in. Can those play-like discussions really claim the title of debate? Language, being, progress, disease, the self, and many more, all simple notions seemingly impossible to define, yet of which we seem to have a never-failing intuition. Why not add debate to that list? And why not process them with that same intrinsic feeling? Rather than aiming for bullet-proof argumentation, speak of what resonates, of what someone might be willing to fight for.

INTERVIEW WITH PROF. CATARINA DUTILH NOVAES

THIAGO J. COCCO ROQUE

ILLUSTRATION: JOSJE VAN DER LAAN

For the first of The Illogician MoL alumni interview series, meet Prof. Catarina Dutilh Novaes (VU). Catarina is a philosopher who moves effortlessly from medieval manuscripts to modern logic, asking big questions about reasoning, dialogue, and what it *really* means to argue well. Hope you enjoy this wonderful conversation with this illustrious MoL alumna. Let us know who you want us to interview next!

QUESTION 1

You were part of the very first cohort of the ILLC's Master of Logic. The program is famous for its interdisciplinary character. Also you recently received the Lakatos Award, which is traditionally awarded to philosophers of science, so it is a big statement to the interdisciplinary nature of your work and your general approach to philosophy. Were you already drawn to this kind of interdisciplinarity before arriving, or was it something that emerged and developed through the ILLC environment itself?

ANSWER 1

So first of all, when I was a high school student, I was interested in everything. So in that sense, this already comes from that time. And, in high school, except for physics, I liked everything. I loved mathematics, but I also really like biology, history, and I even liked chemistry. I did two years of my high school in France, and there I had much more advanced mathematics than I had gotten in Brazil. And this is where my love for mathematics first emerged. And so in that sense, I've always been interdisciplinary in the broader sense of the term.

I then went to study philosophy In Brazil. But while I was studying philosophy, I also took courses in mathematics. Most of my elective courses were in mathematics. Although in Brazil we don't have the major minor system, I usually say I have a major in philosophy and a minor in mathematics. So I already liked this combination of philosophy and mathematics. And then I went to the ILLC because I wanted to study and do research in philosophy of logic, but I felt that I didn't know enough knowledge in logic because when I was in Brazil, I only did a few courses in logic. Then I decided, well, you know, if I really want to be a philosopher of logic, I have to know more logic, and going to the ILLC was the ideal option for me.

First of all, during my time, it was only a one year program, so it was very concentrated. And when I started there, I said, you know what, I want to do the mathematical logic track, since I was there to learn more logic, right? More mathematics and more logic. But then of course, it was very, very hard, very hard. So I did not have the ideal background for it. So I really had to work very hard and I actually even failed one course, which had

never happened to me in my life! So it was a very interesting and sobering experience. So I realized, you know, how little I actually knew. So I worked really hard on catching up. And one of the people who helped me a lot was Dick de Jongh, who was my mentor at the time. We had weekly tutorial sessions where he was helping me catch up with my courses. Towards the end of the year, I figured, you know, I really need to do something more related to philosophy because that's what I'm good at. Just then I started taking philosophy courses. One course in particular that was very formative for me was a course by Martin Stokhof in Philosophy of Language. During one of the first sessions I was like, yes, I know how to do this. This is much easier for me. Martin has become one of my main mentors throughout my career. So that course was. So for me it was very good to be in this interdisciplinary environment and to be able to develop both my mathematical skills, which was what I mostly had hoped to develop, and also the more philosophical side of logic. When it was time to choose the topic for my thesis I decided to dedicate myself fully to philosophy. I did my thesis actually on Medieval Logic. But back in the day nobody at the ILLC knew anything about medieval logic, and when I proposed the topic to Dick de Jongh and Frank Veltman, who was also involved in supervising me, they accepted just because they were curious to see where it would go. And then later I went on to write my PhD also on medieval logic. The fact that I could do this for my masters had a lot of influence also in my career. You know, that was this thing I was interested in back then and my interests have changed now of course, but it's really a combination of all these different experiences that I had that result in this fascinating mix of topics.

QUESTION 2

Now when you apply for the MoL you can find a myriad of information about it. Student experiences, program structure, research opportunities etc. As one of the very first MoL students, you entered the program at a time when it was still new and perhaps a bit mysterious. Did being part of those early cohorts give the experience a special sense of novelty or experimentation? What did you expect the program to be like back then?

ANSWER 2

Even the Internet was a new thing, right? Even the idea that there was information on the Internet for you to find was a relatively new thing, right? But the thing is that I, so my grandparents were Dutch, they were from the Netherlands and they moved to Brazil and my mother was born in Brazil and I was born in Brazil, but we always had a connection with the Netherlands. So I, you know, in a way it was kind of already looking at, for something in the Netherlands, right? And I mean, at the time the MoL was the only Masters of logic available probably in the whole world. So in that sense for me, right, it was a no brainer. To be honest, I don't think I really even consider other options at all. I was just like, okay, this is it! I didn't know much, so it could have gone very, very wrong as well. And I remember that I went to talk to Dick De Jongh, who was the MoL coordinator at the time, to see if he would admit me. In some sense the MoL at that point wasn't very selective because of course it was a new program and they really had to, you know, attract as many interested students as possible. And I think now it's probably much, much more selective. But I mean, I can say that I think looking back, right, it was just one year which is completely insane, right? Because we were writing our thesis during the summer, right? You had courses from September until June and then July, August, you

were writing the thesis. Whereas now it's a two year program, so there's more time to mature, and to really develop as a researcher.

QUESTION 3

If you were to enroll in the MoL program today, which courses would you be most excited, or terrified, to take?

ANSWER 3

Yeah, it's a good question. I mean I would have take a look. For example, some of the people who were my professor back then are still professors now. So one of them I remember was Alexandru Baltag who taught modal logic, and that was a really difficult course. But I did like half of it, so maybe I would finish that course, right. And the other course I would retake, which was very difficult is proof theory. At the time it was thought by Anne Troelstra who's of course a very iconic figure, but he was not very nice to me, to be honest. I mean he generally he had kind of like tough love approach. And also he didn't understand what I was doing there. I mean first of all I was a woman, and second, I wasn't even a mathematician! I did the course, and I passed proof theory. But he was always kind of a little mean to me during the course. However, I must say many years later, when I was awarded a grant, he wrote an email to personally congratulate me. Maybe he thought then that actually his tough love approach had actually helped me. That could be.

QUESTION 4

So with every new MoL cohort every year there are more and more women. And many of them were interested in knowing how was it back in the day, because we now that logic, and academia in general, has been overrepresented by males. So I wanted to ask you if you feel there has been a positive change over the years.

ANSWER 4

This is a great question. One thing is that when I was studying, I was not particularly aware of gender issues. So basically back then, I was not a feminist in the conscious way. And one of the reasons for that was that when I was growing up in Brazil, I had many really great female role models in academia. My mom was an academic. Many of my professors were women. So it never even occurred to me before that, you know, that this was a thing. You know, that's how naive I was. And I only really started like becoming consciously a feminist much, much later. Actually, when I was at the ILLC as a postdoc was when I really started, like, seeing a lot of things that were obviously there, but I wasn't seeing them. But I believe that there's been, of course, over the last 25 years, a huge change in how much this is discussed, right? And how much deserved attention this topic gets. And, I mean, not only in academia, but also, of course, with the MeToo movement and many other societal developments. I mean, of course, feminism had been around for many, many decades, but it was, you know, to some extent, kind of a fringe movement, as it were. And I think one thing that has happened in the last 25 years is that feminism, certainly in philosophy, has become a much more central perspective, which is a good thing.

One of the implications, of course, is us thinking much more about the subtle ways in which academia discourages women from pursuing academic careers and the importance of role models and the role that the academic environment play. If you're the only woman in your cohort, of course that's not definitive, maybe you're fine with that, but most likely it's going to be difficult. And for many, many years in my career, when I was working primarily philosophy of logic, I was very often the only woman in the room. At

some point it really started bothering me, maybe because I started seeing it more.

In fact, one thing I did was, I think it was 2009, I started a list of women working in Logic just to be put online because I at the time there was this blog called Feminist Philosophers and they were often talking about how, like, in many conferences and all speakers are male which sends a very problematic signal. One of the arguments used by organizers was like, ``yeah, but there are no women working on my topic. That's why I'm not inviting them.'' So I thought, well, what if we compile a list of women working in Logic so that whenever this argument comes up, we can just send the list. But more importantly, people who are organizing conferences can have a resource that they can look at, you know, to see, like to look for ideas for women they could invite. These things, I think, have really changed for the better.

And I also want to say that while gender, of course, a very important dimension, it's not the only category of exclusion. There are all kinds of other categories that are relevant. We also need to think about race, we need to think about class, we need to think about people with disabilities, about sexual orientation. I'm just bringing this kind of what we call this intersectional perspective.

QUESTION 5

Because of the longevity of the program, and perhaps the diversity of students and faculty, I feel like the ILLC accumulated some ``legends'' over the years. Do you remember any early ILLC folklore professors, courses, or events that became part of the institute's mythology?

ANSWER 5

I mean, all these people that I've already mentioned. Also more on the philosophy side of things, Martin Stokhof, who I mentioned, Frank Veltman, Jeroen Groenendijk and Michiel van Lambalgen. And well, Alexandru was already around, you know, who's very iconic, has always been and will always be. One person I want to mention, who was very important to me back then was Carlos Areces, who's an Argentinian logician and was a PhD candidate at the ILLC. He was always super helpful and really supportive and he helped me with a lot of stuff.

QUESTION 6

If you put ten MoL students in a room and ask them to define 'logic', how many definitions do you expect, and which one would you pick to escape the room?

ANSWER 6

Twenty. Twenty different definitions. Yes, at least. Because some of them will have more than like three. So I mean, it's gonna be more definitions than people.

QUESTION 7

To finalize, I wanted to ask you what is the most Illogical thing you've done while studying in the MoL?

ANSWER 7

Yeah. Well, I mean, you could say that it was very illogical for me to sign up for the mathematical logic track. I could have done the easier route, which would be to go for the philosophical logic track. So that was, in some sense, quite illogical. But on the other hand, well, it was not illogical in the sense that I learned precisely what, I wanted to learn. So it depends on how you look at it. One very illogical thing that I've done was to bike on top of the tram track and of course, slip and fall. Like, you know, if you think about it, obviously it's not a good idea. It was my first year in Amsterdam. There were many such things that I had to learn the hard way.

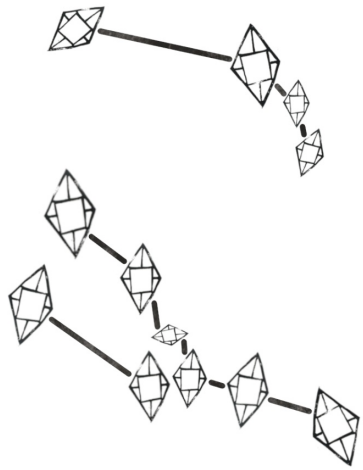
(IL)LOGICAL HOROSCOPE

MATTEO CELLI, JOSJE VAN DER LAAN, BARDO MAIENBORN, GIANNIS RACHMANIS

Need some direction in your life or curious what kind of semester awaits you? Read your (il)logical horoscope!

Aries (Mar 21 – Apr 19)

Are you a linguist? Then, you should feed the ducks. A great semester comes before you with a lot of despair and hope, sadness and happiness, alcohol and water. Feeding the ducks is really important to keep you aligned with yourself and banish the bad energy, particularly when the new moon arises. As your season peaks and the season of Taurus draws near, be careful of structured semantic objects because spring will spring and lambdas will conquer. Be strong, persist and feed the ducks. And do not forget about the rabbits (if you can even see them).

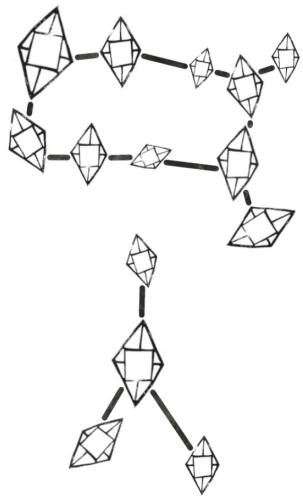


Taurus (Apr 20 – May 20)

Your love for contradictions is getting dangerous. You know what you do not believe and you mean how you do, meaning mean manners matter most. With the Mercury retrograde starting soon, you have to make sure your communication is safe and encrypted, so no one will find out about your weekend adventures at Nikhef. Lovewise, you know what to do, but the result is not trivial. A little reflexive and transitive advice written in the stars: think before you talk and act before you think.

Gemini (May 21 – Jun 20)

With the sun moving to Sagittarius and activating your 9th house, you will have to make decisions about your professional life, your philosophical retrospections and cryptographical inquiries of the meaning of reference. Keep your kitchen tidy after you clean it, so that you have a clear head space. Remember to swing left and not in the middle of your head, otherwise you may end up with an infinite descending chain of unfortunate events. But do not give up yet: things will simplify once you finally learn to accept you will never be fully able to pronounce Grzegorczyk.

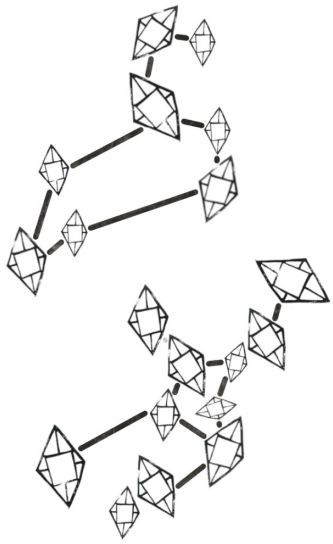


Cancer (Jun 21 – Jul 22)

You always sketch proofs and you procrastinate on checking the details. Beware! With Jupiter in the house of Mars, this can reveal fatal: this planet configuration makes base cases and boolean cases non-trivial, and will make you forget that also a limit case may be needed. For any n proofs by induction, there will always be an n+1-th. The solution, as for any problem, is to move higher-order, to ensure nothing is comprehensible anymore and you can continue procrastinating, as you do best.

Leo (Jul 23 – Aug 22)

Last winter, not everyone got to see Leo dancing around the pole and leaving the entire club with a look on their face as if they had just seen a proof of falsum. Luckily, Mercury slicks in Gemini, Venus flaunts herself in Leo, and Mars - ever the warrior - squares the circle just enough to stir cosmic chaos. As a result, your nights will unfold not with the necessity of becoming a motherducker in SP107, but the possibility of being alpha harron on the hunt for some cheeky snacks from science döner. If you are hungry for more, just add an extra axis to extend the cube and find the nearest path to the fridge.

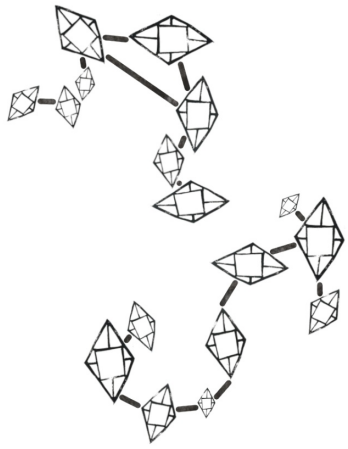


Virgo (Aug 23 – Sept 22)

What is the meaning of life? What is the life of meaning? These questions have been bothering you for weeks now, but with the sun moving to Sagittarius, the answers will be coming to you soon, and you will finally be able to write that philosophy paper you have been putting off. Apart from that, some of your old enemies turn into lovers as you learn to apply them. Together, you can now complex the computationality with your hearts beating in a new algo-rythm (boom, boom boom, boom boom boom...) for the rest of your lifes.

Libra (Sept 23 – Oct 22)

New year, new you! You do not see the point of it anymore, so you decide to go against all odds and solve the problem of induction by induction on the structure of the problem. Now the next step is to move on, and make an epoché about your past choices - but is that even possible after what happened? Just pick up a new hobby, like racebiking. Going for your highest (purple) goals might prove too much to handle, you should settle with what you can feasibly achieve: finitary mathematics.



Scorpio (Oct 23 – Nov 21)

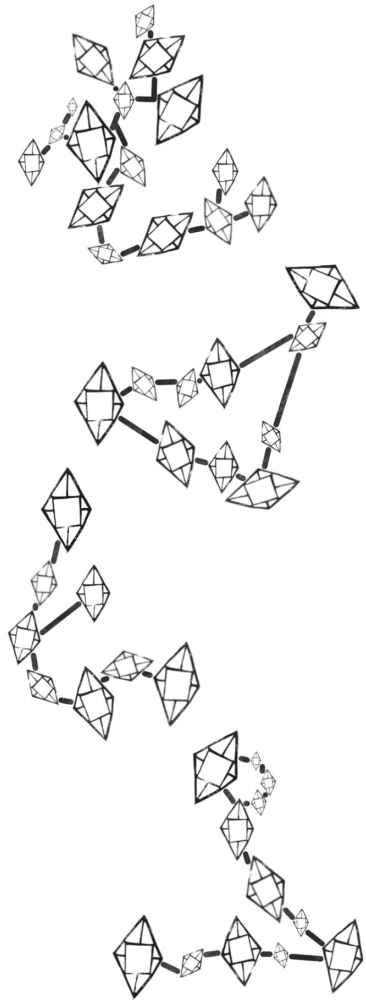
All the star charts commute, and your efforts will come to completion. But also, by duality, to co-completion. Do not be stubborn, and do not see yourself as an object in isolation: individuals are better understood as the collection of maps into them and from them. Therefore try to make an adjoint effort with like-minded, co-laborative people, and trust only those who make a fully faithful and essentially surjective on objects representation of you. Clearly, the dual also holds, proof by chasing the stars.

Sagittarius (Nov 22 – Dec 21)

This horoscope is false, as the stars are giving inconsistent signs: either you will explode, or you adopt a paraconsistent logic to infer a meaningful message from them. It seems like your exam grades will be good and bad, you will and will not find true love, and at work you will and you will not have to take up exceptional (or even not exceptional) responsibilities. Maybe there exists an impossible world in which you are the present king of France, but not even bald. Think about this: what is your most sound inconsistency?

Aquarius (Jan 20 – Feb 18)

In this semester many things you've been wanting to have fixed will come together for you. Theories will amalgamate, models complete and cuts will disappear. Now that the Earth starts its new circle around the sun you can start to play more challenging games. Be careful, though, of the fire, because Paris is still burning. Proofs by pictures give you new purpose in life, when this has been drained out of you in efforts to conquer the non-existent fires in your non-peers.



Capricorn (Dec 22 – Jan 19)

You have a semester of constructivity ahead of you. Avoid doing proofs by contradiction, certain forms of contraposition and elimination of double negations: your love life would be in trouble, and you will only meet people who are not not interested in you, or themselves not uninteresting. Now that you have entered the constructive (orthodox?) Church, maybe it is the initial time to try pineapple on pizza as a method of math destruction.

Pisces (Feb 19 – Mar 20)

A period begins where the water of the sea reaches the sand. At the same time, Mercury retrograde in your 9th forms a triangle with Saturn retrograde in your 1st and plans for studies or travel may return to your mind. The least medium month has passed, but there is still infinite potential (or potential infinite?) to construct a better future for yourself. About love: love is approaching as it moves away - can you catch it in finite time?

ILLUSTRATIONS: HUGO RENNINGS

CRYPTIC CLUES

ESTEL KOOLE, DAVID KÜHNEMANN, BARDO MAIENBORN, MARIANA RIO COSTA

Cryptic clues originated in the early 20th century Britain as an evolution of regular crossword puzzles. These crosswords are called cryptic for the fact that the clues include some kind of wordplay or hidden meaning, and that the answer usually does not match the plain reading of the clue. This (often more difficult) variety of crosswords gained popularity in the UK and beyond, with cryptic crosswords appearing regularly in major newspapers like The Times and The Guardian.

In their modern format, cryptic clues generally include the following:

1. A definition: Found at the beginning or end of the clue, the plain reading of this part by itself will describe the answer to the puzzle, just as a clue in a regular crossword. However, this description will often only match the answer in a broader, humorous, or more roundabout way and hence the definition itself is often not enough to solve the puzzle.
2. Wordplay: The other part of the clue will form some sort of wordplay that, when solved, also yields the answer to the puzzle. The wordplay itself can usually itself be divided into wordplay indicator, words that hint at what kind of wordplay needs to be applied, and fodder, i.e. words on which the wordplay acts.

Example

Single tutors accidentally fish in freshwater. (5)

Here, the wordplay part of the clue is single tutors accidentally. The first indicator, single, acts on the fodder, tutors, and tells us to take not the plural but the singular form of tutors, i.e. tutor. Secondly, "accidentally" is an anagram inidicator, telling us to rearrange the letters of tutor to find the answer. (Imagine the letters getting into an accident and being disordered as a result.) Doing so gives us trout, an answer that matches the definition part of the clue, fish in freshwater.

Clues

In the following you find two logic themed cryptic clues.

HE IS GASLIGHTING THEIR MULES WITHOUT REST (6)

A COLOURING THAT IS SOUND IN ALL POSSIBLE WORLDS (4)

ILLC
WORD SEARCH

FIND THE (IL)LOGICAL WORDS!

L	I	N	G	U	I	S	T	I	C	S	P
M	E	E	F	F	O	C	N	N	N	C	H
P	I	A	I	M	Z	I	O	T	A	L	I
M	R	D	R	L	C	C	I	U	I	L	L
A	R	L	S	K	U	I	T	I	C	I	O
D	M	A	T	H	E	M	A	T	I	C	S
R	F	D	-	L	F	E	T	I	G	D	O
E	A	O	O	J	E	T	U	O	O	D	P
T	M	M	R	O	H	S	P	N	L	V	H
S	I	R	D	X	K	I	M	I	L	M	Y
M	L	L	E	R	I	P	O	S	I	Q	V
A	Y	F	R	A	N	E	C	T	E	O	K

AMSTERDAM
COFFEE
COMPUTATION
EPISTEMIC
FAMILY
FIRST-ORDER
ILLC
ILLOGICIAN
INTUITIONIST
LINGUISTICS
LOGIC
MATHEMATICS
MODAL
MOL
MPML
NICK
NIKHEF
PHILOSOPHY