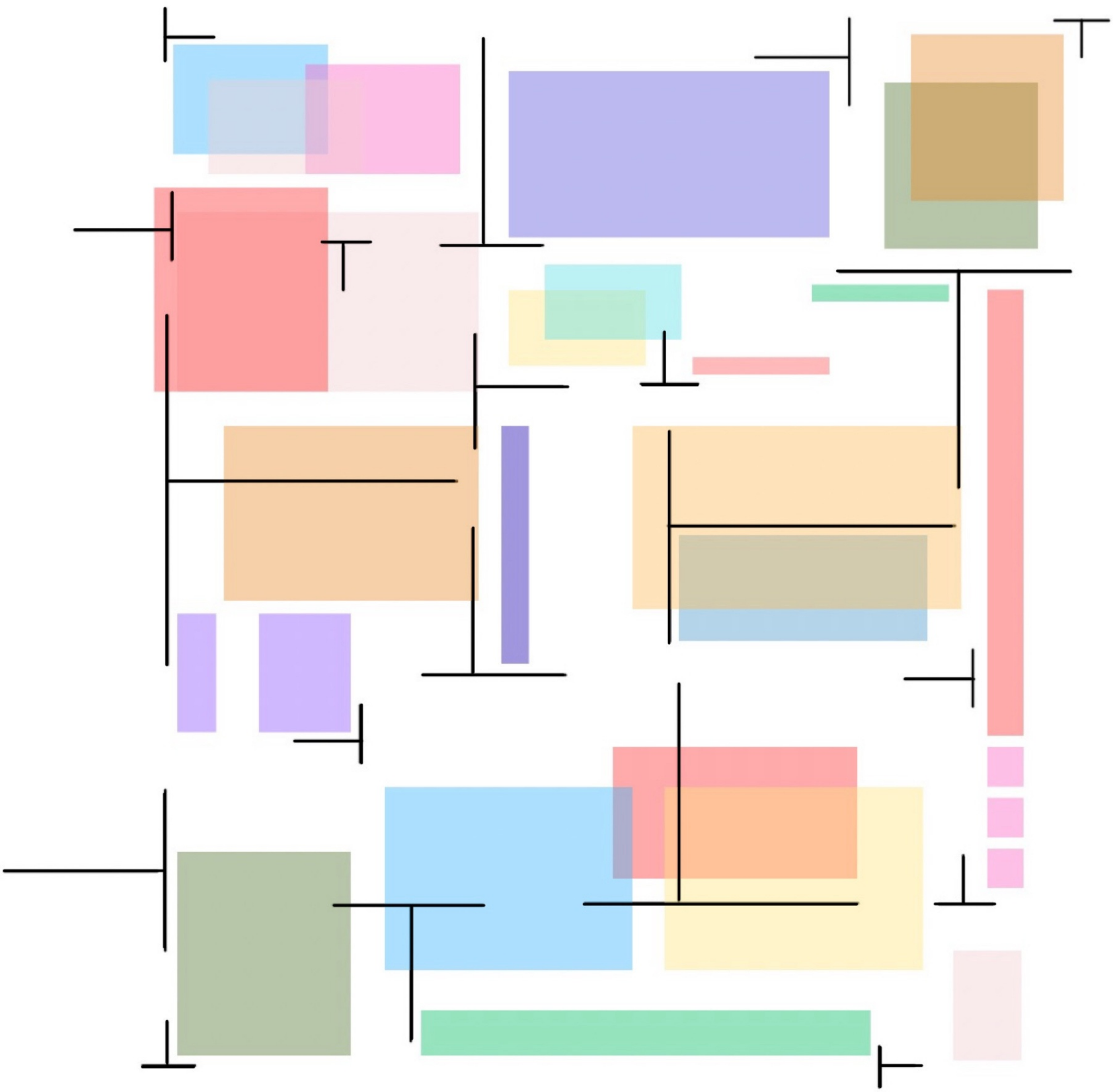
THE ILLOGICIAN



FALL 2025

EDITORIAL

THIAGO COCCO ROQUE | EDITOR-IN-CHIEF

The beauty of the ILLC, and what makes it such a special place, is the diversity of thought. Students and professors from so many different backgrounds and intellectual interests, sharing ideas and passions, all drawn by a single common interest: logic, whatever that means. To celebrate this kaleidoscope of perspectives, we the students of the Master of Logic, decided it was about time for us to have the means to share with the MoL community what we are the most enthusiastic about.

Thus, the Illogician was born. This is the first edition (with hopefully many more to come) of a safe space for students to write essays, jokes, opinions, satires, and whatever else makes their eyes glow within the strange and wonderful universe of Logic.

The reader should expect a true mosaic of articles: from an investigation in elementary topos theory, to a comparison between logic and pizza toppings, we have it all! The final result turn out to be a wholesome reflection of what the MoL is all about: a bunch of people who love talking about Logic, even if no one agrees on what Logic is in the first place.

I hope the reader has as much fun turning the pages as we had creating them!

THE ILLOGICIAN

Fall 2025

Issue 1

Print Run: 120 copies

The Illogician is a semiannual, semiserious periodical specialised in the areas of logic, language, and computation. It is published by Master of Logic students at the Institute for Logic, Language, and Computation of the University of Amsterdam.

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TABLE OF CONTENTS

MATHEMATICIANS, LIST YOUR INGREDIENTS!

JOSJE VAN DER LAAN | MATHEMATICS, PHILOSOPHY

A wise friend of mine once compared being a constructivist to being a vegetarian: even if you are convinced it is the right choice, it is hard to fully commit to it in practice. As a vegetarian and a (developing) constructivist myself, I have come to appreciate just how fitting that comparison is. Both philosophies involve conscious decisions not to use certain things, coming from a deeper commitment to a way of thinking. Just as vegetarians avoid meat, constructivists avoid non-constructive principles like the Law of the Excluded Middle or the Axiom of Choice.[1] Over the years, it has become easier and easier to follow a vegetarian diet and eating meat is (in certain circles) no longer the norm. Perhaps this analogy can be used to our advantage: what we can learn from this development? How could we make it easier to be a constructivist in a world where one often assumes everyone eats "meat" by default?

One small but transformative change that made the life of a vegetarian easier was the rise of labeling. Suddenly, you no longer had to interrogate the waiter or read through ingredient lists with a magnifying glass — a small green "V" quietly told you what was safe to eat. The system did not require anyone to stop eating meat; it simply acknowledged that not everyone does. In mathematics, we are not quite there yet. Most results are served without a list of logical ingredients. Some assumptions, like the Axiom of Choice, tend to come with a warning label, but others, like the Law of the Excluded Middle, are folded so deeply into the dough of standard reasoning that they are rarely mentioned at all.

For the *omnivores* in this world, this is unproblematic, since they do not have to watch what they eat. However, for those who follow a constructive diet, this makes the landscape hard to navigate. One may only realize a proof contains meat when it is already on their plate — provided it has not been so thoroughly processed that you can no longer tell. So, maybe we can take inspiration from the transparency movement in the food industry and start labeling our results; not to restrict, but to inform. Not only would it be helpful to constructivists, it would give any mathematician more insight into what depends on what, and remind all of us that logical assumptions are not neutral background noise: they shape what can be said, and how.

Of course, not everyone is aware of the assumptions underlying their proofs — largely because we are not trained to recognize them. But that very invisibility is part of the problem: how can we be precise in our reasoning if we are unaware of the tools we rely on? As mathematicians, we value precision, so it only makes sense to be exact about the very foundations we build upon.

1. In this analogy, veganism naturally corresponds to intuitionism: a stricter, more demanding version, often met with polite confusion at dinner parties.

Once we start listing our ingredients, a natural next step follows: we begin to wonder which recipes succeed without certain controversial components. We try reworking them, testing whether the dish remains just as satisfying when prepared with alternative methods. This does not only benefit constructivists — it enriches our collective mathematical cuisine. Sometimes, we discover that the meat was never essential. Other times, it proves so central that removing it transforms the result entirely. But even then, the exercise teaches us something important: apparently, this dish is unavailable to vegetarians. On the bright side, when we do manage to find a constructive proof of a formerly classical result, the dish becomes one that everyone can enjoy. Classical mathematicians can still add the Law of the Excluded Middle to taste — maybe to enrich the flavor of the result — but the base recipe is now accessible to all.

If we care about foundations — and as logicians, I certainly hope that we do — then we should care about transparency. So perhaps it is time we start treating our proofs a little more like recipes. Let us label them as being constructive or not, adapt them, and occasionally rethink them from a position that steps away from the norms - after all, history shows us that what is common practice is not always the most desirable or enlightened way forward. If we are transparent about our ingredients, everyone, regardless of their diet, knows exactly what they are being served. And who knows? Maybe one day, using excluded middle in a proof will raise as many eyebrows as pineapple on pizza — an unexpected ingredient that sparks a heated debate.

THE ILLOGICIANS' CONVERSATION

VALENTIN RICHARD | LINGUISTICS

In this crazy discussion, each speech turn contains a semantically deviant or infelicitous utterance. Can you guess what is causing each oddity?

B: Hi Gottlob! In what way are you doing?

G: Yes! What about you, Bertrand?

B: I'm very happy! Yesterday, I came back from a trip to France or Paris.

B: Most of the master students came with me, that is, all of them. It was their first time in Paris.

G: Really? It surprises me, who has ever been to Paris.

B: After that trip, every student stopped hating Paris. Only one student used to hate Paris.

B: We went to a restaurant. We ate a French ratatouille. Then we went to the Louvre museum. It was delicious!

G: Colorless green ratatouilles always sleep deliciously.

B: Some students took a dessert. They ate democracy.

G: Great! And how was the Louvre? Did you learn that fact about the painting *Mona Lisa?*: It is Lisa Gherardini that Francesco del Giocondo ordered a painting that portrays.

B: I did! This is such a famous piece. If no student took a selfie in front of this painting, it will probably be published online.

G: Here is a fun fact you might have missed, though. Five of the six Da Vinci paintings exhibited in the Louvre are made of oil on a panel. It is made of chalk on paper. Can you guess which one it is?

B: It is not. *Mona Lisa* and it might be *Mona Lisa*.

G: The solution is: DA VINCI made *Portrait of Isabella d'Este* with chalk on paper, not someone else.

B: Oh, I didn't know that. More people like Da Vinci than I do. Do you like art history?

G: I once read a book about gender equality. By the way, where was your hotel?

B: On earth.

G: Nice! But how much did you not pay for all of this?

B: A lot! The university didn't give every student any money for the trip.

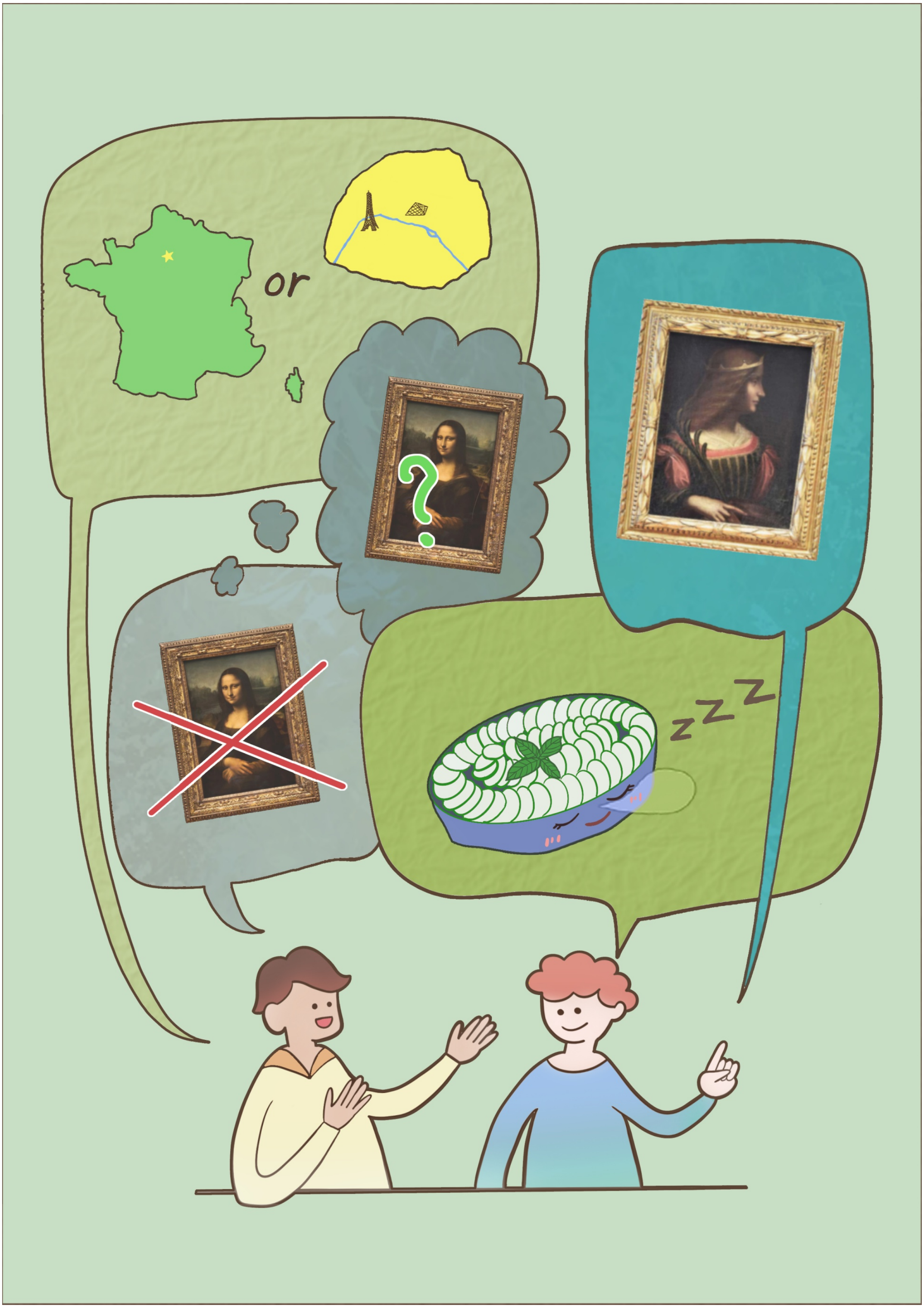
G: At least, in France you may pay in euros or in pounds. So you didn't pay change fees. Did you take the time to get informed about French politics?

B: I did! It is false that the king of France is bald. And some ministers look authoritarian.

G: Be careful what you say! Any French student might enter this room. He disagrees with you.

B: Right! I will go back to my office and be as mute as a ringing fire alarm. See you!

The answers can be found at https://valentin-d-richard.fr/Misc/illogician.pdf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MERKLE PUZZLES: HOW TO TELL A SECRET

DAVID KÜHNEMANN | COMPUTATION

It is Christmas 1974, in the midst of the cold war, and intelligence officer Alice is trying to reach her asset Bob, who is undercover in Budapest, from her apartment in Amsterdam. She knows Bob has confidential information to share with her, but the two of them suspect their telephone line may be tapped. Indeed, East German counter-intelligence agent Eve is listening in to their conversation. The problem of securely transmitting information over an insecure channel has been studied for thousands of years and could, for many purposes, be considered a solved problem by the 1970s. In particular, ciphers like the *one-time pad* allowed for unbreakable encryption of communication as long as the two communicating parties agreed on a large enough shared secret, the so-called *secret key*, ahead of time. Unfortunately for our two protagonists, Bob lost the secret key he had agreed with Alice during his latest undercover operation. Up until this point in history, conventional wisdom was that Alice and Bob were now screwed: That without a secret key shared ahead of time, there was no way for Bob to transmit information such that Alice could decipher it but eavesdropping Eve could not.

Luckily for Alice, just a few months before on the other side of the world, UC Berkeley undergraduate student Ralph Merkle wrote a course project proposal that challenged this long-standing belief (Merkle 1974). His idea, nowadays known as *Merkle puzzles*, consituted the world's first *key exchange protocol*.

KEY EXCHANGE PROTOCOLS

A *key exchange protocol* is a set of instructions telling Alice and Bob what to compute and send over the channel, such that, after the execution of all instructions, both obtain the same secret key which is oblivious to any eavesdropper like Eve. Concretely, we parameterize the effort it takes Alice and Bob to execute the protocol by a *security parameter* n. (This could for example be the total number of steps they need to execute, but we'll see a better quantity soon.) We then measure the amount of effort it would take Eve to have any real chance at recovering the secret Alice and Bob agreed to just from the information sent via the channel in terms of the same security parameter n. A key exchange protocol requires that Eve needs to exert strictly more effort than Alice and Bob. A protocol is more secure the larger the gap between these two is. Once Alice and Bob have agreed on a shared secret, they can use the existing cryptographic protocols to communicate securely.

ASYMMETRY IN COMPUTATION

Merkle's breakthrough result relies on an observation about asymmetry in the world of computation, specifically that we think some functions f are easy to compute for any given input x but hard to invert. Breaking up a completed jig-saw puzzle does not lose any information in the sense that most jig-saw puzzles have only one sensible arrangement, but clearly the process of breaking up the completed puzzle is much easier than putting the individual puzzle pieces back together. Some mathematical problems seem to be of a similar nature. For example: Multiplying prime numbers together is easy, while we don't know any efficient procedure to split composite numbers into all of their prime factors. The notion of non-invertibility needed for Merkle puzzles is as follows: We want some easy-to-compute function f, such that given a (uniformly) random input x, no algorithm can successfully determine an x' such that $f(x) = f(x')$ but with a negligible probability. There are many candidate functions that are believed to exhibit this behavior, but actually proving this would have wide-ranging implications (including that $P \neq NP$) and seems beyond our current methods. Instead, for analyzing a protocol like this we turn to an idealized form of such a function, a so-called *Random Oracle (RO)*. Think of a RO as a single magic box that each of Alice, Bob, and Eve have access to and can query with an input x. The first time the RO is queried on the input x, it picks a random value y to output. Every subsequent time the RO is queried with the same x it outputs the same y. It is easy to see that such a RO is hard-to-invert, since given a y there is clearly no better strategy of finding the corresponding input x than simply trying out all possible inputs. Of course, random oracles don't actually exist in the real world, but in practise we can instantiate (substitute) them with any of the candidate hard-to-invert functions.[1]

MERKLE PUZZLES

Let $n \in \mathbb{N}$ be the security parameter and RO: $\{1, \ldots, n^2\} \rightarrow \{0, 1\}^n$ be a random oracle that maps inputs between 1 and n^2 to binary strings of length n. The Merkle Puzzle key exchange now proceeds as follows:[2]

1. Alice chooses $10n$ numbers $x_1, \ldots, x_{10n}$ uniformly at random from the range between 1 and n^2. For each x_i she queries the RO and receives an output $a_i = RO(x_i)$. She sends all $10n$ values $a_1, \ldots, a_{10n}$ to Bob via the channel.
2. Analogously, Bob chooses $10n$ numbers $y_1, \ldots, y_{10n}$ uniformly at random from the range between 1 and n^2. For each y_i he queries the RO and receives an output $b_i = RO(x_i)$. He sends all $b_1, \ldots, b_{10n}$ over the channel.
3. If there exists a pair (i, j) such that $a_i = b_j$, then both Alice and Bob take the lexicographically smallest (first) such pair and take x_i and y_j as their secret keys respectively. We call such a pair a *collision*. If no such collision exists, they both take 1 as their secret key.

In our model, the security parameter n therefore quantifies how many times Alice and Bob have to query the RO ($10n$ times each). This is a good measure as computing the hard-to-invert function which the RO stands in for is usually the most computationally intensive part of the protocol by far. Alice and Bob will agree to the same (non-trivial) key if:

1. the RO is injective, whereby we ensure any collision in step 3 actually leads to Alice and Bob's adopted keys x_i and y_j being equal, and
2. a collision occurs.

The chance that the random oracle assigns two different inputs x_1 and x_2 the same output y is

1. There are (highly contrived) cryptographic protocols for which this step goes wrong (Canetti, Goldreich, and Halevi 2004), but this is not an issue here.

2. We actually look at the more formal description of Merkle Puzzles given by (Barak and Mahmoody-Ghidary 2017).

$1/2^n$. By a union bound, the likelihood of this not happening for any of the $\approx n^2 \cdot n^2$ pairs of distinct inputs is at least $1 - n^4/2^n$ which quickly approaches 100% as we increase the security parameter n.

THE BIRTHDAY PROBLEM

As for point 2, one might intuitively think that because Alice and Bob only pick $10n$ random values from a possible range of n^2 (which is far larger for even moderate choices of n) there is a good chance of no collision occurring at all. To see why this is intuition is wrong, let us consider the closely relate *birthday problem*. How large does a friend group have to be for at least two people in the group to share the same birthday? Despite the probability of two people sharing the same birthday being just $1/365$, it turns out that with just 23 people, it is more likely than not that two of them do in fact share the same birthday. Intuitively, this is because we have to consider all possible pairs of people, of which there are $(23 \cdot 2)/22 = 253$ within a group of 23. Applying this to our collision problem, the chance of no collision happening turns out to be less than 1 in 2^{100} (for n at least 10). Therefore, Alice and Bob fail to agree on a shared secret key with only a miniscule probability.

NO LUCK FOR THE EAVESDROPPER

What about Eve, who has been listening in this whole time? All the information she sees communicated are the a_is and b_is. She can of course also find the collision $a_i = b_j$ but the only way she can compute the underlying shared secret is to invert the random oracle given the output $a_i = b_j$. As mentioned earlier, there can be no better strategy than just trying out all n^2 possible inputs, and we expect her to find the correct input only after $n^2/2$ tries. As we increase the security parameter n, this effort soon exceeds the $10n$ queries needed by Alice and Bob, meaning Merkle Puzzles indeed form a key exchange.

OUTLOOK

The "square" gap between the effort of magnitude n by Alice and Bob against the $\approx n^2$ required by Eve to recover the shared secret is not considered large enough for practical use. Luckily, only two years later in 1976, Whitfield Diffie and Martin Hellman (whom Merkle had approached after his course instructor rejected his project idea) developed an improved key exchange with a much larger gap in effort (n vs $\approx 2^{\sqrt[3]{n}}$), using not any hard-to-invert function, but *one specific candidate* and its special algebraic properties (Diffie and Hellman 1976). More than 40 years after they were conceived, Boaz Barak and Mohammad Mahmoody proved that Merkle Puzzles are in fact optimally secure among key exchanges that work with arbitrary hard-to-invert functions (Barak and Mahmoody-Ghidary 2017).

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ROTATING SHAPES: THE LONG(EST) LINE

WHY IS IT SO LONG AND WHY CAN'T WE MAKE IT LONGER?

MAX WEHMEIER | MATHEMATICS

"The long line is a topological space somewhat similar to the real line, but in a certain sense ›longer‹."[1]

To understand this statement, we will firstly do a short recap of topology. A lot of statements will just be given, but the reader is encouraged to verify that these hold.

TOPOLOGY-BASICS

A *topological space* is a tuple $(X, \mathscr{T})$ consisting of a non-empty set X and a topology $\mathscr{T}$ on X, i.e. $\mathscr{T}$ is a collection of subsets of X, that contains $\varnothing$ and X and is closed under binary intersections and arbitrary unions. We call the elements of $\mathscr{T}$ the *open sets* in X.

When $\mathscr{T}$ is understood, we sometimes refer to the whole space just by X. We call a collection $\mathscr{B} \subseteq \mathscr{T}$ a *basis* for $\mathscr{T}$ if every element of $\mathscr{T}$ is the union of elements in $\mathscr{B}$. In this case, we say that $\mathscr{B}$ *generates* the topology.

Similarly, every set $\mathscr{B} \subseteq \mathscr{P}(X)$, which is closed under intersections and covers X (i.e. $\bigcup\mathscr{B} = X$) forms a basis for some topology.

For example, if we have a linearly ordered space $(X, <)$ with at least two elements, then we can define an interval as the set of all elements between two given points. The collection of all intervals forms a basis for the *order topology* on X. This topology will play an important role for the long line.

REVISITING $\mathbb{R}_+$

Before we look at the long line, we will construct the long ray as a preliminary step. To make the construction easier to understand, we will first look at a different way to view the topological space of non-negative real numbers $\mathbb{R}_+$ (including 0). Note that the usual topology on $\mathbb{R}_+$ is just the order topology given by the usual ordering.

One can easily see that this basis as described earlier is uncountable. However, since the rationals are dense in the reals, we can restrict a

1. Cf. https://en.wikipedia.org/wiki/Long_line_(topology)

and b to be rationals and would obtain the same topology.

This means that $\mathbb{R}_+$ is *second countable*, i.e. has a countable basis. Furthermore, this makes it first countable: Every point has a countable neighborhood basis.
Given a point $x \in X$, we call a collection $\mathscr{B}_x \subseteq \mathscr{T}$ of opens containing x a *neighborhood basis* if for every open set U containing x, there is a $V \in \mathscr{B}_x$, such that $V \subseteq U$. So by considering

$$\mathscr{B}_x := \{V \in \mathscr{B} : x \in V\},$$

we see that second countable spaces are also first countable. Instead of viewing $\mathbb{R}_+$ as the subset of $\mathbb{R}$ where all values are greater or equal to 0, we can also construct it via

$$\mathbb{R}'_+ := \mathbb{N} \times [0, 1).$$

This has a nice intuition: Since every element is a pair (n, r) with a natural number n and a decimal r, we can view it as the number $n + r$.

Moreover, if we give it the lexicographical ordering by setting $(n, r) < (m, q)$ iff $n < m$ or $n = m$ and $r < q$, it becomes order-isomorphic to $\mathbb{R}_+$ with the usual order. Thus we get the usual topological space if we equip it with the order topology. Technically, it is only *homeomorphic* to the original space, but since homeomorphisms are the isomorphisms in topology, we will consider them to be the same.
You might notice that instead of $\mathbb{N}$, we could have written the ordinal ω.

SO WHAT IF WE USED DIFFERENT ORDINALS?

For finite ordinals (apart from 0), we essentially get the intervals $[0, n)$, which are homeomorphic to $[0, 1)$ by division by n. Using the fact that $[0, 1) \cong \mathbb{R}_+$, one can inductively show that for all countable ordinals the result is still homeomorphic to $\mathbb{R}_+$. But what happens after that?

CONSTRUCTING L+

The smallest uncountable ordinal is ω_1.
If you are unfamiliar with ordinals, you can imagine the natural numbers, but there are uncountably many of them.
By saying that they are like natural numbers, we want to emphasize that every number has a unique successor. For a deeper digression, we refer to (Levy 2012).
This way, we can define

$$L_+ := \omega_1 \times [0, 1).$$

Giving it the lexicographical ordering and the order topology yields the *long ray*.

Firstly, let us look at why the long ray is a line, i.e. in what sense is it similar to $\mathbb{R}_+$. As we will see it does not globally look like $\mathbb{R}_+$, so it is not homeomorphic to it.
However, it is locally homeomorphic to it. Every point has a neighborhood which is homeomorphic to $\mathbb{R}_+$. Indeed, if $(\alpha, r) \in L_+$, then α must be a countable ordinal. So $\alpha + 1$ is still countable and

$$(\alpha, r) \in U := (\alpha + 1) \times [0, 1).$$

Since $\alpha + 1$ is also countable, we already discussed that U must be homeomorphic to $\mathbb{R}_+$, so it is the neighborhood we were looking for.

Because local homeomorphisms preserve first countability, we get that L_+ is first countable.

PROPERTIES OF L_+

In $\mathbb{R}_+$, there are sequences like $a_n := n$ that do not converge. This can almost not happen in L_+: While alternating sequences still do not converge, every sequence has a convergent subsequence, which topologists call *sequential compactness*.
For any sequence $(a_n)_{n \in \mathbb{N}}$ in L_+, we know that

$$a_n = (\alpha_n, r_n)$$

for all n with all α_n countable. Thus the supremum of them must be a countable ordinal α.

So all α_n are smaller than a:= $\underline{\alpha+1}$. (I use the notation $\underline{\beta}$:= (β,0) to improve readability.) Thus $a_n \in [0,a]$$ for all n.

We have already seen that $[\underline{0},a) := (\alpha+1) \times [0,1) \cong [0,1)$, so it should not be hard to convince yourself that $[$\underline{0}$,a] \cong [0,1]$. Since the latter is sequentially compact, our sequence must converge. This means that the long ray is too long for sequences to diverge. Now we can also conclude that L_+ cannot be homeomorphic to $\mathbb{R}_+$: if we had

$$f\colon \mathbb{R}_+ \cong L_+,$$

then instead of calculating the limit of $a_n := n$ in $\mathbb{R}_+$, we could calculate the limit of $b_n := f(a_n)$ in L_+.

If $b := \lim b_n$ in L_+, then $a := f^{-1}(b)$ would be the limit of a_n in $\mathbb{R}_+$. However, we know that this sequence does not converge in $\mathbb{R}_+$. Thus they cannot be homeomorphic.

I already hinted at the fact that the long line is in a sense the "longest" a line can be. What I mean by that is that spaces constructed via

$$L_+^\kappa := \kappa \times [0,1)$$

with an ordinal $\kappa > \omega_1$ fail to be locally homeomorphic to $\mathbb{R}_+$. And if they do not even locally look like $\mathbb{R}_+$, in what sense is that space still a line?

If L_+^κ were locally homeomorphic to $\mathbb{R}_+$, then there must be a neighborhood U of $\underline{\omega_1} \in L_+^\kappa$ and a homeomorphism $f\colon U \cong \mathbb{R}_+$. Since $\mathbb{R}_+$ is path-connected and $\underline{0} \in U$, there is a path γ from $f(\underline{0})$ to $f(\underline{\omega_1})$.

Since $f^{-1} \circ \gamma$ is continuous, [0, 1] is compact, L_+^κ is Hausdorff and we can assume that γ is bijective, by (Munkres 2000, Th. 26.6) this would

$$L_+ \subseteq [\underline{0}, \underline{\omega_1}] \subseteq (f^{-1} \circ \gamma)([0,1]),$$

mean that $f^{-1} \circ \gamma$ is a homeomorphism. But since the long ray would be homeomorphic to some subset of [0, 1]. This must be of the form [0, p) for p ∈ (0, 1].

But since [0, p) is homeomorphic to $\mathbb{R}_+$, we would get $L_+ \cong \mathbb{R}_+$. But we concluded earlier this cannot happen, so there cannot be any path from $\underline{0}$ to $\underline{\omega_1}$. After adding just one more copy of [0, 1) to L_+, it fails to be locally homeomorphic to $\mathbb{R}_+$. So the long ray is indeed the longest a ray can be.

THE LONG LINE

So far, we have only talked about the long ray, although the article is supposed to be about the long line. It is rather easy to get the long line from two long rays: We can just glue them together at $\underline{0}$.

$$L := (L_+ \sqcup L_+)/\sim,$$

Formally we can define this as
where ~ is the smallest equivalence relation identifying both zeros, but nothing more. The intuition for this is literally taking two long rays and glueing them together at $\underline{0}$ with one pointing into the "positive" direction and the other in the "negative" direction.

We have seen many interesting properties of L_+, so which ones does the long line have? Basically all of them!

This is because the subspace topology on L_+ given by L is just the topology we were considering the whole time on it. In a similar way as for the long ray, we can see that the long line is locally homeomorphic to ℝ (since it now does not have a smallest element). But it cannot be homeomorphic to ℝ, because then the long ray would have to be homeomorphic to $\mathbb{R}_+$. So it is first countable, but cannot be second countable (otherwise the long ray would have to be second countable as well).
It is also sequentially compact by a very similar argument. So we can just claim all these results now because we did the work earlier.

A MODIFIED CLASSIFICATION THEOREM

The topology-minded people might remember the classification theorem of connected 1-manifolds. It states that up to homeomorphism the only connected 1-manifolds are ℝ and the circle S^1. Intuitively, connected 1-manifolds are topological spaces that are locally homeomorphic to ℝ and satisfy some additional criteria.
You might notice that the long line is locally homeomorphic to ℝ, but not homeomorphic to either ℝ or S^1, so it is not covered by this theorem. This is because manifolds are required to be second countable.

After dropping this condition, there are up to homeomorphism four connected 1-manifolds: ℝ, S^1, L and the half-long line, which is L_+ glued together with $\mathbb{R}_+$ (Frolík 1962).

So the long line does not only serve as a funny counterexample, but plays an important role in the classification of topological spaces.

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INGREDIENTS AND LOGIC: IS PUTTING PINEAPPLE ON PIZZA RATIONAL?

GIUSEPPE MANES | PHILOSOPHY, COMPUTATION

A MOL DINNER

After a week of intense study, four MoL students - E, M, S, and J - met at a renowned Amsterdam pizzeria. The restaurant offered a special deal:

Any group of students ordering pizza from the menu (excluding pineapple) would get 10% off and free dessert.

The chef, B, considered pineapple to be the worst topping and explained that it is a very bad combination in terms of flavors, and also does not correspond to traditional standards. So, he deliberately excluded it from the deal in order to convince people not to order it. The menu is divided in two sections *Traditional* and *Non-Traditional*. Apart from the Pineapple Pizza, all the other ones in the menu are valid for the deal, therefore the students can choose freely. Furthermore, we notice that the number of *Traditional* and *Non-Traditional* pizzas is the same, excluding the pineapple one.

All the students being Logicians, they think of themselves as rational agents, in the *Bayesian* sense, so each of them has a coherent initial distribution, i.e. their credence distribution over their belief is a probabilistic distribution. In particular, we have that given the set of toppings they assign their preference corresponding to the belief in sentences of the form

I like the pizza (with) A.

respecting *Kolmogorov's Axioms*.

We call the sentences of this type φ_A, where A is the flavor of the pizza. In order to study the distribution we will use the credence function $cr : \varphi_A \to [0, 1]$. The distributions will be the following:

- E is a *food anarchist* and loves pizza with strange flavors like Natto[1] (that unfortunately is not on the menu), and would prefer to eat a pizza with no conventional topping instead of the "boring" traditional ones. Therefore, she will have a distribution in the sentences of the type φ as follows, where A is the section of *Traditional Pizzas* in the menu and B is the section of *Non-Traditional Pizzas*:
  - $cr(\varphi_{Natto}) = 0.5$
  - $cr(\varphi_{b \in B}) = 0.3/|B|$
  - $cr(\varphi_{a \in A}) = 0.2/|A|$
- M loves Sauerkraut pizza (not on the menu) since it was a pizza that his mom always cooked for him when he was a child. He also loves Italy and traditional cuisine so he prefers traditional ingredients in his pizzas. Therefore, he will have a distribution in the sentences of the type φ as follows:
  - $cr(\varphi_{Sauerkraut}) = 0.4$
  - $cr(\varphi_{a \in A}) = 0.4/|A|$
  - $cr(\varphi_{b \in B}) = 0.1/|B|$
- S is an open-minded guy that loves traditional food but also likes experimenting, and he has a slight preference for traditional tastes to

1. Natto is a traditional Japanese food made from soybeans that have been fermented.

which he is used to but doesn't mind in principle to try new food. Therefore, he will have a distribution in the sentences of the type φ as follows:

- $cr(\varphi_{a \in A}) = 0.6/|A|$
- $cr(\varphi_{b \in B}) = 0.4/|B|$

- J is an elegant and sophisticated person, and she loves flavors combinations in this pizzeria, of which she is a common client. The only pizzas that she will not eat are the pizzas with meat since she is a vegetarian. Therefore, she will have a distribution in the sentences of the type φ as follows:
  - $cr(\varphi_{NoMeat}) = 1/|NoMeat|$

BAYESIAN PRINCIPLES AND IRRATIONALITY MEASURES

According to the Bayesian framework we are using, we have that an agent should accord their credence to the credence of an expert. This law is called the *Expert Principle* (Elga 2007), which is formally expressed by the following conditional probability

$$cr(\varphi \mid cr_E(\varphi) = x) = x,$$

where cr_E is the credence distribution of the expert. This principle is one of the many rationality constraints possible in the Bayesian framework.

Now, let's return to our group of students. We'll now present an example to show how these kinds of principles can be applied within the theory.

Let our Logicians update their credence distribution according to the credence of our expert, namely the chef B. All of them will align their credence on the proposition $\varphi_{Pineapple}$ with the credenceof the chef in it, that is 0, leaving the other propositions in the same way. This updating lead our Logicians to behave as irrational agents: by the Bayesian Probabilistic framework (i.e. the Bayesian epistemological account with the least amount of principles that an agent should respect to be rational, namely only *Kolmogorov's Axioms*) the agent always needs to have normalized credences distributions. In our example this means that each agent needs to have credences such that $cr(\varphi_{Margherita}) + cr(\varphi_{Figs}) + cr(\varphi_{Pineapple}) = 1$ and all the credences have to be more non-negative real values (just by a simple application of Kolmogorov Axioms). If our agents don't normalize their credence in the update process, we have that they cannot be coherent and, therefore irrational.

Now, the question that we raise and that we are interested to investigate is the following:

Which of our agents is the least irrational?

This is equivalent to asking which of our agents is closer to a coherent distribution.

In order to study this matter, we consider the theory exposed in (Staffel 2019) where the general aim is to construct a theoretical framework that can compute the distance of an irrational agent to the closest coherent position. We will not consider the whole theory, but only a *naïve* version of it, where we take as our distance measure the Absolute Distance between a point of a rational distribution and a point of an irrational distribution, which is also

2. We are assuming that this set is finite.

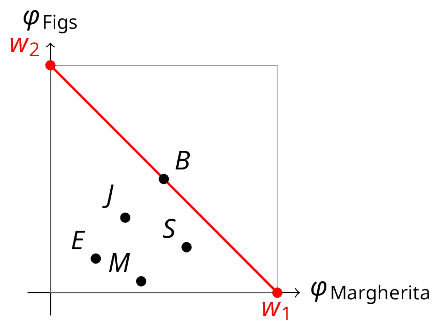
argued in (Staffel 2019) to be the best one for this purpose.

More formally, each agent has a set of credences $\mathscr{B} = \{\varphi_1, \dots, \varphi_n\}$[2] for some $n \in \mathbb{N}$, and we can represent this set with a point in a n-dimensional Cartesian Space. Then, we will uniquely assign each agent to a point and, based on principles of probability, we can construct the region of our space that corresponds to a rational distribution. Then using the Absolute Distance we can calculate the minimum distance between the points representing the irrational credences distributions of the agents. We can also optimize the region selected using Scores, like Brier's or the Absolute one (Titelbaum 2022), which are distance measures between the credences and the possible worlds (in order to do this we also need some minimization in the process).

We can see an example as follows: Consider our set $A = \{\varphi_{\text{Margherita}}\}$ and $B = \{\varphi_{\text{Pineapple}}, \varphi_{\text{Figs}}\}$. Accordingly we have that our distributions will be the following after the update

- E will have a credence of 0.2 in $\varphi_{\text{Margherita}}$ and 0.15 in φ_{Figs} (Pizza Natto is not in the menu so we will not consider it)
- M will have a credence of 0.4 in $\varphi_{\text{Margherita}}$ and 0.05 in φ_{Figs} (Pizza Sauerkraut is not in the menu so we will not consider it)
- S will have a credence of 0.6 in $\varphi_{\text{Margherita}}$ and 0.2 in φ_{Figs}
- J will have a credence of 1/3 both in $\varphi_{\text{Margherita}}$ and φ_{Figs}

Applying our further constraint we have that $cr(\varphi_{\text{Pineapple}}) = 0$. Since all our agents have a 0 credence on $\varphi_{\text{Pineapple}}$, as they update according to the *Expert Principle*, we can consider a plane to represent the measures.



It is easy to see that any rational distribution (with all the constraints involved) has to be a point in the line $y = -x + 1$ for $x \in [0, 1]$. Then minimizing the Absolute Score (which we choose in order to use a same distance measure) we obtain that the best point is the uniform distribution between the two propositions, i.e. the point $B = (0.5, 0.5)$ in our plane. Then doing some calculation we can easily see that the closest one to minimize his credence is J. Also, S is really close to get a coherent distribution, if we don't consider the Score, but it wouldn't be the optimal distribution given all the possible worlds (in our case 2 since we must choose a pizza and only one pizza, so one of them has to be true).

COMPUTATIONAL COST OF THE BAYESIAN FRAMEWORK

This framework is really nice, but there is an issue given by the computational cost of Bayesian epistemology that grows with the number of principles that we consider, and the number of propositions in the input (thus the number of possible worlds). Also, considering the simple case presented in this article, one can easily imagine that computing the most rational agent, when both the number of students and the number of pizzas increase, becomes demanding from a computational point of view. In fact, a lot of different algorithms have been presented using different types of measures that grow exponentially with the input size. So our last question is: *Can a real agent realistically use this theory to guide their reasoning in daily situations?*

In (Kwisthout, Wareham, and Van Rooij 2011), this computational cost has been studied, and furthermore has been evidenced that a lot of problems in the field can be proven to be NP-complete problems. A problem of this kind is the Most Probable Explanation (MPE). Here, given a set of hypotheses and a set of observed pieces of evidence, along with a probabilistic model that describes how these are related (such as a Bayesian network), the task is to determine which truth assignment to the hypotheses is most likely to be correct, given the observed evidence. In other words, we are looking for the overall combination of truth values for the hypotheses that has the highest conditional probability according to the model.

Algorithms of the kind that we presented here, and more generally in all Bayesian Epistemology, can be proven to be NP-complete as well. This kind of task can quickly become too demanding for a real agent, because the computational complexity of the underlying algorithm grows exponentially with the amount of information involved. In other words, as the number of hypotheses and observations increases, finding the most probable assignment becomes increasingly hard from a computational point of view.

Therefore, while this theory offers a comprehensive framework for assessing both rationality and irrationality, its computational demands make it impractical for use by real agents.

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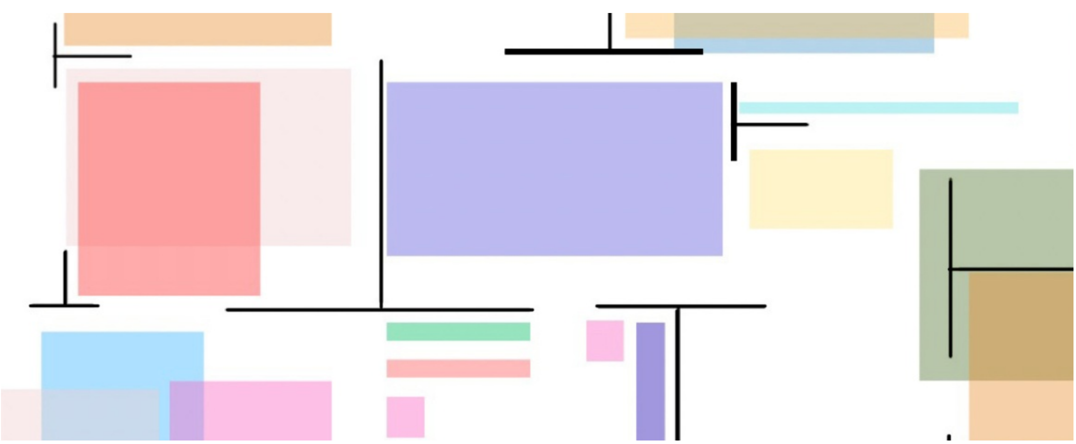

WHEN "IF" COMES AFTER "THEN"

THOMAS VAN DER LEER | LINGUISTICS

Suppose you want to tell someone that you believe that the first edition of *The Illogician* will be a success, provided that the articles in it are well written. You could convey this message in the usual way, by uttering a sentence such as (1).

(1) If the articles are well written, *The Illogician* will be a success.

However, it would not be strange to instead say (or hear) exactly the same sentence, but in a different order:

(2) *The Illogician* will be a success, if the articles are well written.

We thus see variation in the clause order of conditional sentences. The canonical order is visible in (1), where the antecedent precedes the consequent, whereas the conditional in (2) is 'non-canonically' ordered: here, the consequent precedes the antecedent.

The existence of this variation is, I think, quite interesting. After all, nothing changes to the literal meaning of (1) if you for some reason decide to formulate it as (2). Why, then, do we sometimes decide to utter conditionals in a different order, or do we at least find this decision acceptable? What makes us prefer, in certain situations, the non-canonical order over the canonical one?

CONNECTEDNESS

As with many linguistic phenomena, it is very likely that the answer to this intriguing question is not one-sided. More factors are probably in play, influencing us to prefer one order over the other. One such factor, for instance, could have to do with the different social functions conditionals can carry out. For example, a conditional could be used to make a prediction. We saw an example of such a predictive conditional in (1) and (2): in the consequent, it is mentioned what would happen (*The Illogician* becoming a success) given the event described in the antecedent takes place (the articles being well written).

A completely different way of using conditionals is illustrated in sentence (3) – an example taken from (Austin 1970, 212). What seems to be the case in these often called 'biscuit conditionals' is that the antecedent is used to make a comment on the speech act performed in the consequent, in the case of (3) the offering of biscuits.

(3) There are biscuits on the sideboard if you want them.

While predictive and biscuit conditionals are just two instances of the many social functions a conditional can fulfil, they illustrate an important way in which these functions can diverge. Whereas predictive conditionals such as (1) imply a clear causal or temporal connection between their antecedent and consequent, biscuit conditionals such as (3) do not – the biscuits are on the sideboard, whether you want them or not. Let us call this, following (Reuneker 2022), a difference in their *connectedness*.

So, what then would be the prediction regarding clause order preference based on this 'connectedness' feature conditionals may or may not possess? When there is a strong connection present between the clauses of a conditional, it is conceivable that this connection incites a desire to place the clauses in a specific order. Take predictive conditionals: since the event described in the antecedent precedes the event of the consequent in a temporal and causal way, the most natural thing to do as a speaker seems to be to order the sentential clauses correspondingly. In fact, research shows that a mismatch between the temporal and clause order of a conditional often results in a response time penalty for listeners (Evans and Newstead 1977).

Conversely, when a conditional lacks connectedness, a preference for one specific clause order becomes much less obvious, if not completely absent. We saw biscuit conditionals as an example of non-connected conditionals: the antecedent merely comments on what is being said in the consequent. One can imagine how it would not matter much where or when this commenting takes place. For instance, (4) and (5) seem equally natural:

(4) If you're thirsty, there is beer in the fridge.
(5) There is beer in the fridge, if you're thirsty.

Thus, according to the hypothesis described above, conditionals high in connectedness receive a clear preference for one of the clause orders – the canonical in the case of predictive conditionals – while those low in connectedness do not.

GIVEN AND NEW

Connectedness is, however, only one possible factor influencing our clause order preferences. A very different one discussed in the literature looks at the information structure of the discourse in which the conditional is uttered, and more specifically at the so-called given-new principle. According to this principle, we prefer a sentence to first present information that is given in or inferable from the context, if it includes any. Only after this would we like the sentence to introduce information that is completely new. For example, if we are talking about how I struggled for the exam of Modal Logic, and I would like to say to you that 1) I almost failed the exam but that 2) Mary easily passed, the preferred thing for me to do is to utter those two propositions in that order, not the other way around. The given-new principle has been very successful in explaining how discourses are structured and has received a lot of empirical support (see e.g. Clark and Haviland 1977; Haviland and Clark 1974).

It is hopefully easy to see how this principle can be applied to the topic of discussion. If a conditional is uttered in a discourse and the information in the antecedent relates much more directly to what has been said before than the information in the consequent, the given-new principle makes the prediction that we would prefer the canonical order. If the roles are reversed and the consequent, not the antecedent, contains the information that connects the conditional to the previous context, we would expect to observe a stronger preference for the non-canonical order.

Of course, there are many more factors one could think of that possibly influence our preferences for conditional clause ordering. For instance, the order seems to depend not only on the relative amount of words present in the antecedent or the consequent (see e.g. Diessel 2005; Ford and Thompson 1986), but also on the mode (spoken vs written) in which the conditional appears (Reuneker 2020).

AN EXPERIMENTAL INVESTIGATION

In an experimental study I carried out with Lotte Hogeweg (Radboud University), we put both the connectedness and the given-new hypothesis to the test (Van der Leer and Hogeweg 2024). We presented Dutch predictive and biscuit conditionals embedded in a small context to native speakers, asking them to indicate their preference for clause order on a slider. Interestingly, we found confirmation for both hypotheses. Even more interestingly though, we also found a general preference for the canonical order across all conditions. Since the non-canonical order does occur in natural language, it is likely that there are factors next to connectedness and information structure that language users take into account when deciding on a preference for clause ordering.

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WHICH (IL)LOGICIAN ARE YOU?

HISTORY IS FULL OF BRILLIANT (AND BAFFLING) THINKERS. WHICH LEGENDARY (IL)LOGICIAN MATCHES YOUR MIND? TAKE THE QUIZ TO FIND OUT!

HOW DO YOU CONFESS YOUR LOVE?

A. "I would prove to you how much I love you, if only I could."

B. "Let us build something real together."

C. "No more *ignorabimus* in love!"

D. "You are the prime mover behind all that I do."

E. "For you, I'd accept a logical paradox."

F. "My love for you is like the natural numbers: actually infinite."

WHAT ROLE DO YOU PLAY IN A GROUP PROJECT?

A. You are the annoying one, who disproves conjectures.

B. You do not trust anyone and do everything yourself.

C. You make very ambitious plans.

D. You are the bossy one, and people take notes for you.

E. You make sure no one oversteps each other, and prefer hierarchical roles.

F. You go mental and do not believe in what you wrote.

HOW DO YOU REACT TO A BREAKUP?

A. You go to a stripclub and find true love.

B. You get along the best with yourself anyway.

C. You admit your faults in the relationship.

D. After all, you were opposites.

E. You quickly find someone else, who is more consistent.

F. You are in denial: you see it, but you don't believe it.

HOW DO YOU REACT TO FINDING A GREAT PROOF TO AN OPEN PROBLEM?

A. Fame is not for you: you become afraid of being poisoned.

B. You actually spend more time looking for unsolvable ones.

C. You want to solve them all, time to move on.

D. What open problems? You already solved everything.

E. You are the one who found the problem.

F. You text your bestie, but are perplexed.

WHO ARE YOU AT A PARTY?

A. Socially anxious, you don't touch anything at the buffet.

B. Quietly in your own corner, observing.

C. The one giving a motivational speech, inspiring everyone to reach their potential.

D. Directing the conversations, making sure everyone stays on topic.

E. The person calmly resolving social conflicts behind the scenes.

F. You are the one fascinated by the endless flow of people.

WHAT IS YOUR FAVOURITE SONG?

A. "The Sound of Silence" - Simon & Garfunkel

B. "My Way" - Frank Sinatra

C. "Harder, Better, Faster, Stronger" - Daft Punk

D. "Do I Wanna Know?" - Arctic Monkeys

E. "The Logical Song" - Supertramp

F. "I Don't Trust Myself (With Loving You)" - John Mayer

MOSTLY A: GÖDEL

Smart and introverted, but with unexpected sides and interests. Your mom may not like your partner, but without them, you are incomplete. You have a flair for limitative results, and the boldest conjectures always go through the unforgiving lens of your intellect. You are brutally honest, and not afraid of shattering one's dreams (especially those of C people): if something cannot be proven, you have elegant ways of showing them.

MOSTLY B: BROUWER

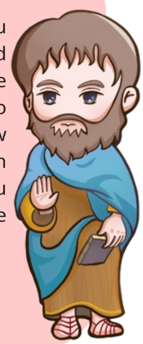
You do not believe in anything without proof, and you often do not believe in anyone either: you want things done your way, and you are sceptical of principles like the excluded middle, that appeal to "how things are". For you, mathematics is an art well beyond dry formalism: it is part of life, impossible to disentangle from philosophy.

MOSTLY C: HILBERT

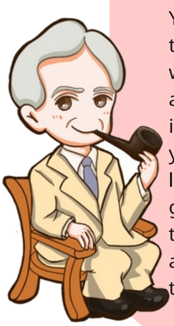
You are ambitious and optimistic, and you set bold goals. Even if your plans do not end up as desired, you still manage to get something good out of it. You do not like leaving questions unanswered, and you believe that you should start from the simple foundational issues to disentangle them.

MOSTLY D: ARISTOTLE

You are systematic and you like to understand and classify everything. You are also a pioneer, and hope to be associated with a new field of inquiry. You like when people listen to you, and you don't mind if someone else takes notes on your behalf.



MOSTLY E: RUSSELL



You are a radical progressive thinker, but you are still a dandy who loves enjoying life. You are ambitious in your ideas, and there is no dogma that will stand against your eye for paradoxes, which are likely to shake commonly accepted grounds. To solve them, you think that everything in logic has a place, and we should not mix different things up.

MOSTLY F: CANTOR

You prove beautiful theorems, but you are wary of your own results: you see them, but you do not believe them. How is it even possible that they are true? Don't go crazy about it, your besties agree that you are onto something.

REGULARITY PROPERTIES OF THE REALS AND THE AXIOM OF

YIPU LI | PHILOSOPHY, MATHEMATICS

In this article, we aim to give an introduction to the set theoretic study of relation between regularity properties of the reals and the Axiom of Choice. Broadly conceived, regularity properties are desirable properties of set of real that appear in various different fields in mathematics. Among them, we are going to sketch *Lebesgue Measurability* (LM) and the *Ramsey Property* (RP). Diverse and seemingly unrelated they are, it turns out that they share a surprisingly similar pattern. We are going to show that with Axiom of Choice, we can show the existence of sets that break LM and RP respectively.

Moreover, due to a celebrated result by Robert M. Solovay (Solovay, 1970), using the method of forcing, we are able to show that the full Axiom of Choice is necessary to break these regularity properties. The negative result was proved by constructing a model where all usual of Zermelo–Fraenkel axioms hold and Axiom of Choice fails, yet ALL sets of reals have LM and RP.

THE AXIOM OF CHOICE

The Axiom of Choice states:

Axiom of Choice: *For any family* A *of pairwise disjoint nonempty sets, there exists a choice function* f *such that* f(A) ∈ A.

Russell famously gave an analogy to help understand the axiom (Math Stack Exchange contributors 2017). Say a millionare owns infinitely many pairs of shoes: Shoes_i, $i \in I$. Then there is always a way to choose one shoe out of each pair: you may choose the left shoe out of each pair, since the left shoe and the right one are different. The function is described by $\text{Shoes}_i \mapsto$ the only element s in S_i s.t. s is the left one. The Axiom of Choice is not involved here.

However, the case for socks, Socks_i, $i \in I$, is different. It is conceivable that the socks for left and right feet are indistinguishable, so there is no specific rule you can apply to choose a sock out of each pair. This illustrates where the Axiom of Choice comes into play. The Axiom asserts that nevertheless, there is a function that chooses from each pair: there is f s.t. $f(\text{Socks}_i) \in \text{Socks}_i$.

REGULARITY PROPERTIES

Lebesgue Measurability (LM)

Lebesgue measurability is one of the most crucial concept in analysis and probability theory. Rather than giving a detailed account of the theory, we are satisfied with informally saying that a set $X \subseteq \mathbb{R}$ is Lebesgue measurable means that it is regular in the sense that the result of measuring its size from the outside is equal to measuring its size from the inside (The outer measure of X is equal to its inner measure). In this sense, a LM set cannot be too chaotic.

In 1905, with the help of Axiom of Choice, Giuseppe Vitali (Vitali, 1905) showed that there is a set that is not LM. Such sets are now known as the Vitali sets.

Ramsey Property (RP)

In combinatorial, the infinite Ramsey theorem states that any infinite binary graph contains an infinite homogeneous set, a set whose elements are either pairwise connected or pairwise disconnected. Indeed, the theorem is true for any n-ary graphs, where edges connect n elements instead of 2.

Thinking beyond the finite case, we can ask whether the similar strutural pattern emerges for graphs with infinite aried edges. For a set X, let $[X]^\omega$ denote the set of all subsets of X of size ω.

Definition: Let $\mathscr{R} \subseteq [\mathbb{N}]^\omega$ be a set of infinite subsets of $\mathbb{N}$. $\mathscr{R}$ is said to have the Ramsey Property if there exists an infinite homogeneous set $A \subseteq \mathbb{N}$, i.e. either $[A]^\omega \subseteq \mathscr{R}$ or $[A]^\omega \cap \mathscr{R} = \varnothing$.

A set $\mathscr{R} \subseteq [\mathbb{N}]^\omega$ is thus conceived as the infinite aried edges of a infinite graph on underlying set $\mathbb{N}$, and is said to have RP if there is a subgraph that is totally connected or totally disconnected.

The property does not seem outright to be a property of sets of real numbers, but rather a property of subsets of $[\mathbb{N}]^\omega$. However, from a set theoretic point of view, $[\mathbb{N}]^\omega$ and $\mathbb{R}$ are of course equinumerous and there is a way to view $[\mathbb{N}]^\omega$ as sharing similar properties as the real line $\mathbb{R}$. So for now we indulge ourselves in calling RP a regular property of the set of reals.

And you guessed it, as the Axiom of Choice comes into play, we can show the existence of a set failing RP.

Proposition (AC): *There exists a set without the Ramsey Property.*

Here is the idea of the proof. We consider the equivalence relation $\equiv_{fin}$ on $[\mathbb{N}]^\omega$ by $X \equiv_{fin} Y$ iff their symmetric difference is finite.

By the Axiom of Choice, for each equivalent class of the relation we can pick a represantative. Consider the function f that takes an equivalent class [X] to an element in it, $f([X]) \in [X]$. Now we consider the following set $\mathscr{S} \subseteq [\mathbb{N}]^\omega$:

$$\mathscr{S} := \{X \in [\mathbb{N}]^\omega : |X \Delta f([X])| \text{ is even}\}.$$

We invite the readers to check that the set $\mathscr{S}$ cannot have RP, basically because the property $|X \Delta f([X])|$ is even can be made to fail by altering X finitely, but which equivalence class X belongs to does not change so easily.

THE SOLOVAY MODEL AND FULL REGULARITY

In our above discussion, the standard story is that there is a desired regularity property which we wish it to be true for all sets of the reals, yet the Axiom of Choice comes into play and ruins our day.

Hence it is natural to ask whether Axiom of Choice is necessary for the existence of these irregular sets. Is it possible to prove the failure of LM and RP on some sets without using the Axiom? In 1970, Solovay answered the question negatively for LM and later Mathias (Mathias, 1977) showed that Solovay's argument works for RP too.

Solovay's theorem states that assuming that ZFC, the Zermelo–Fraenkel Axioms with Choice is consistent with some large cardinal assumption, then

Theorem (Solovay–Mathias). *It is consistent with ZF that all sets of reals are Lebesgue measurable and have the Ramsey Property.*

Let's forget about the part of the assumption that mentions large cardinal assumption, which is not a key point for our topic. We have to assume the consistency of ZFC basically because by Gödel's second incompleteness theorem, we can never prove that ZFC is consistent.

The method with which Solovay established the result was forcing, allowing one to start from a model of set theory and construct a new one. By starting from a model of ZFC and some large cardinal assumption, Solovay constructed a new model with a submodel where AC fails, yet all sets have LM and RP. This submodel was known as the Solovay model.

Perhaps more surprisingly, in the Solovay model the principle of Dependent Choice, a marginally weaker version of the Axiom of choice holds. This fact indicates that that existence of pathological sets assumes truly the full power of AC. This perhaps serves as an argument against full AC, depending on our philosophical view.

Moreover, the story for LM and RP is happening again and again. There are other regularity properties discussed in various field of mathematics: perfect set property in set theory; the property of Baire in topology and etcetera. Under AC, the aspiring regularity property of sets of the reals does not hold on all sets of the reals. On the other hand, AC is necessary for the existence of such pathological sets, as all these regular properties hold for all sets of the reals in the Solovay model.

Wrapping things up, in this article we peaked into an 'ideal world' where regularity prevails on the realm of reals, and where the Axiom of Choice does not distort things and produce chaotic objects that does not have the notion of size, does not have a homogeneous part, etcetera.

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THE LOGICO-EMPIRICAL FOUNDATIONS OF MEDIEVAL WITCHCRAFT ASCERTAINMENT: A PYTHONIAN ANALYSIS

THIAGO COCCO ROQUE | PHILOSOPHY

This paper examines the application of proto-scientific methodology in the well-documented medieval inquiry portrayed in *Monty Python and the Holy Grail*, specifically the "She's a Witch!" scene. At the core of the tribunal's reasoning lies the Duck-Weight Equivalence Principle (DWEP), which posits that any individual whose mass precisely matches that of an avian specimen (specifically, *Anas platyrhynchos*) must, by ancient epistemic law, consist of the same material as ducks. Since ducks float and wood floats, the DWEP formally asserts that such individuals are composed of wood, and hence, by transitive ontological inference, must be witches. This leads directly to the Lemma of Buoyant Malfeasance, though objections persist among anti-floatation theorists who deny the ontological reality of ducks.

Let $\mathbb{H}$ denote the category of peasants, $\mathbb{D}$ the space of floating waterfowl (modulo plumage), and $\mu : \mathbb{H} \sqcup \mathbb{D} \to \mathfrak{M}$ a mass-functor valued in the Medieval Reals $\mathfrak{M}$ (cf. the Holy Manual of Weighing Things). We define the **Duck-Weight Equivalence Principle (DWEP)** as:

$$\forall h \in \mathbb{H},\ \exists d \in \mathbb{D} \text{ such that}$$
$$\mu(h) \sim_\epsilon \mu(d) \Rightarrow \mathbf{W}(h)$$

where $\sim_\epsilon$ denotes approximate equivalence under the canonical Feather-Norm topology, and $W : \mathbb{H} \to \mathbb{B}$ is the witch-indicator functor taking values in the Boolean topos $\mathbb{B} = \{\top, \bot\}$. The duck is assumed to be ideal, frictionless, and morally neutral.

From DWEP we derive the **Compositional Parity Lemma (CPL)**:

$$\mu(h) = \mu(d) \Rightarrow \text{Wood}(h)$$

$$\text{Wood}(h) \otimes \text{Burns}(h) \Rightarrow \mathbf{W}(h)$$

where ⊗ denotes the mob-conjunction operator (see *The Standard Model of Angry Peasants*), and Burns(h) is defined iff h is subjected to $\mathscr{F}_{\text{ire}}$, the folklore ignition functor. Attempts to formally invert ⊗ remain obstructed by the Peasant Uncertainty Principle, which states that no angry mob can simultaneously know what it wants *and* why.

The weighing procedure is implemented via a symmetric seesaw morphism

$$\mathscr{S} : \mathbb{H} \times \mathbb{D} \to \textbf{Tilt}$$

constrained by the **Peasant Uncertainty Principle**:

$$\Delta\text{Reason} \cdot \Delta\text{Yelling} \geq \hbar_{\text{turnip}}$$

Experimental results yield:

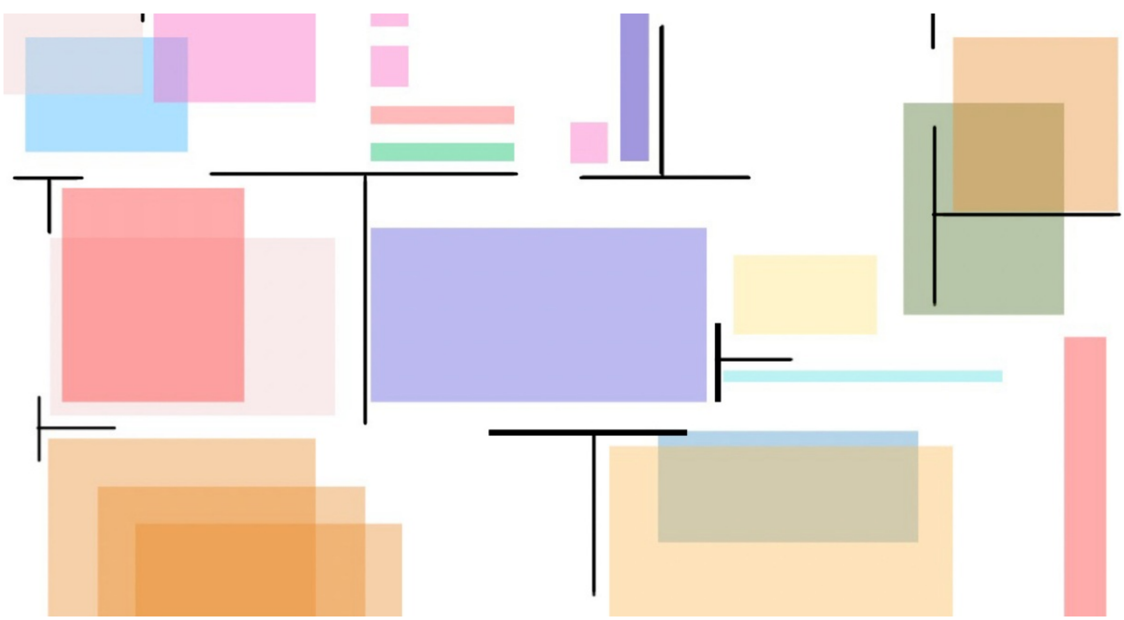
$$\mu(h^*) \cong \mu(d^*) \mod \delta, \quad \delta < \text{bias}_{\text{duckfoot}}$$

By the **Witch Affirmation Theorem (WAT)**:

$$\mu(h) = \mu(d) \Rightarrow \mathbf{W}(h)$$

The proof of WAT is left as an exercise to the reader.

This result consolidates DWEP as a foundational principle in the broader theory of medieval verificationism. While further research is required to determine whether other small animals (e.g., badgers, newts) yield equivalent detection frameworks, the duck-based criterion remains the most parsimonious known method. Applications to legal theory, theology, and buoyancy studies are immediate and far-reaching.



MATHEMATICAL STRUCTURES IN PROGRAMMING LANGUAGES

STEFANO VOLPE | COMPUTATION, MATHEMATICS

If you enjoyed the *Mathematical Structures in Logic* course from last semester, you are going to love Nick's rebranding for the new year: *Mathematical Structures in Programming Languages*! Alright, you got me, I haven't managed to convince Nick yet. Still, don't you find sticking exclusively to logics kind of limiting? There are so many more formal languages out there. What do their semantics look like? What kind of dualities between those shall emerge? Here, let's stick to a very simple programming language, Hutton's razor, defined by the following grammar.

$$L ::= \star\mathbb{N} | L \oplus L$$

Observe that specifying a grammar is akin to presenting a signature Σ (in the sense of first order logic, and of universal algebra): we are stating that ⋆ represents an ℕ-indexed family of 0-ary operators, i.e. constants (L does not appear next to it), and that ⊕ is a binary operator. We can think of Σ as a *"syntax" functor* mapping each possible choice of a domain (in first order logic, or carrier in universal algebra) to the type of "ingredients" needed to construct a program using our grammar.

$$\Sigma(X) = \mathbb{N} \uplus X \times X$$

The above can be read as "You can construct programs by either specifying a natural number (and using ⋆) or by specifying two programs (and combining them with ⊕)".

DENOTATIONAL AND OPERATIONAL SEMANTICS

A logician would probably conclude that the semantics of this language should be given by a function $[\cdot] : L \to \mathbb{N}$ like the following.

$$[x] = \begin{cases} n & \text{if } x = \star n \\ [t_1] + [t_2] & \text{if } x = [t_1 \oplus t_2] \end{cases}$$

As this function is expressing what object each program is meant to denote, we call this kind of semantics *denotational*. Let's spell out the mental process here: a domain, ℕ, was chosen, and our semantics have been specified as a recursively defined function from our grammar to said domain. Equivalently, this map $\Sigma(X) \to X$ is an algebra for the signature mentioned above (in the universal algebra sense, or term-level semantics for our first order theory): we fixed a carrier, ℕ, an ℕ-indexed family of constants, which are specified by the base case, and a binary operator, which is the natural numbers sum operator, as given by the inductive case. Visually, evaluating such a structurally recursive function (algebra) looks like folding the input in on itself until we obtain the result. So **denotational semantics are algebras for the syntax functor**. We use these algebras to know *how* to fold into a single value from our semantic domain. So, much like logics, programming languages also have algebraic semantics! The operation of folding according to our algebra is itself a computable function translating programs written in a high level language (the object language given by our syntax) into a low level language (the metalanguage). Computer scientists call such translators *compilers*.

Believe it or not, programming languages can be so tricky that coming up with this kind of nice compositional semantics is, to date, still quite difficult, and requires learning hard maths. To avoid doing so, computer scientists come up with a proof system instead, and lazily defer the semantics problem to derivability in such a system. This is the same approach as proof-theoretic semantics, which can save you from learning hard maths, as well. In the case of our programming language, the judgements of the proof system will be pairs whose left member will be a program t. On the right of the arrow, the second element will either be a natural number n (indicating that t cannot be simplified any further, and that its result is n), or a second program u (meaning that t reduces to u after one step of computation). In each rule, we use t, u, v as program variables, and n, m as natural number variables.

```
                t ↠ n   u ↠ m
------------   ------------
   ⋆n ↠ n      t⊕u ↠ ⋆(n+m)

   t ↠ u          t ↠ u
------------   -----------
 t⊕v ↠ u⊕v      v⊕t ↠ v⊕u
```

The binary relation $\rightsquigarrow \subseteq L \times (\mathbb{N} \uplus L)$ of all pairs derivable in the proof system describes what kind of operations a machine can perform to manipulate our program, so we refer to these semantics as *operational*. In fact, $\rightsquigarrow$ can be represented as the function $\rightsquigarrow[\cdot] : L \to \mathbb{N} \uplus \mathscr{P}_{fin}L$ mapping irreducible programs to their result, and reducible programs to the collection of possible immediately successive execution steps. To us,

$$b(X) = \mathbb{N} \uplus \mathscr{P}_{\text{fin}}(X)$$

is the *"behaviour" functor* of our programming language. That is, it describes how programs behave at runtime: they either return a result or reduce non-deterministically, but still with finite branching. We called functions of type $\Sigma X \to X$ for some X "algebras" over syntax functor Σ. Hence, it is only natural to call all functions that, like $\rightsquigarrow[\cdot]$, have type $X \to b(X)$ for some X *coalgebras* over behaviour functor b. Visually, any of these coalgebras tells us how to unfold an initial program into a trace of possible execution paths. Hence, **operational semantics are coalgebras for the behaviour functor**. We use them to know "how" to unfold a program repeatedly until (hopefully) we terminate and obtain the collection of all its possible output values. If we call f the operation of repeatedly unfolding according to the $\rightsquigarrow[\cdot]$ coalgebra, we have, for example:

$$\begin{aligned}
&f((\star 3 \oplus \star 2) \oplus (\star 1 \oplus \star 0))\\
&= \{f(\star 5 \oplus (\star 1 \oplus \star 0)), f((\star 3 \oplus \star 2) \oplus \star 1)\}\\
&= \{\{f(\star 5 \oplus \star 1)\}\}\\
&= \{\{\{f(\star 6)\}\}\}\\
&= \{\{\{6\}\}\}
\end{aligned}$$

This is a trace, or execution tree, for our program (think of the syntax tree of {{{6}}}).

Because of how b is defined, its internal nodes are not labelled. In addition, duplicated branches are not allowed, so this trace is conflating the two possible reductions chains. At any rate, this time around, the carrier of our coalgebras is not semantical (the denotational domain), but rather syntactical: it is the language itself! Unfolding programs from such a language into their traces using the specified coalgebra is akin to a procedure simulating their execution step-by-step. Computer scientists call such programs *interpreters*. The duality between denotational and operational semantics is now apparent. A gentler exposition on this is given by (Hutton 2023).

FULL ABSTRACTION

Syntax, algebraic semantics, and coalgebraic semantics: our triptych is now complete. To relate $[\cdot]$ and $\rightsquigarrow [\cdot]$, we introduce a property known as *full abstraction*. A function defined using our language as domain is fully abstract if it maps two programs to the same object just in case such programs are unfolded to the same traces by our operational semantics. Ideally, we would want folding using our denotational semantics to be fully abstract. This would guarantee our denotational semantics are abstract enough to equate two programs with the same traces, but still informative enough to map programs with different traces to different objects. As a simple exercise, can you prove that full abstraction does not hold between $[\cdot]$ and $\rightsquigarrow [\cdot]$?

DISTRIBUTIVITY LAWS

Ever been worried of coming up with mismatching algebraic and topological semantics for your logic, and then wasting time trying to prove a representation theorem that does not hold? With programming languages, *distributivity laws* (Paviotti and Wu 2023) can help us mitigate the analogous problem for denotational and operational semantics, while providing a satisfying bird's-eye view on our duality. These laws describe how our syntax functor Σ distributes over the behaviour functor b. They are parametric in an arbitrary type X, and take the form of a family of functions

$$(\lambda_X : \Sigma(bX) \longrightarrow b(\Sigma X))_{X:\text{Type}}$$

specifying how our syntax interacts with our behaviour. In particular, by defining how Σ can be "pushed past" b, these functions are witnessing how behaviour behaves compositionally with reference to our syntax: behaviour for a program can be obtained composing the ones of the immediate subprograms. Note that our language L, being inductively defined using Σ, is the smallest object closed under it, i.e. the least fixpoint μΣ. So what b-coalgebra of type $\mu\Sigma \rightarrow b(\mu\Sigma)$ shall act as our operational semantics? Because the domain of the desired function is our language, the result is, unsurprisingly, a fold. Specifically, the fold using the following Σ-algebra.

$$b(\text{In}) \circ \lambda_{\mu\Sigma} : \Sigma(b(\mu\Sigma)) \longrightarrow b(\mu\Sigma)$$

What is going on here? Using a fold over a Σ-algebra means we are trying to define our operational semantics recursively on the syntax of the program. The recursive clauses of this definition will be defined componing two operations. First, we use distributivity to compose behaviours for the immediate subterms into a single behaviour for a "standalone" object storing such subterms.

Now, $\text{In} : \Sigma(\mu\Sigma) \to \mu\Sigma$ is a constructor for our inductively defined language. Under normal conditions, it would take the standalone object mentioned above as its input and produce a program. Our second and last step is lifting In using the action on maps of functor b so we can have this construction happen at the behaviour level. The algebra in now complete. Isn't it funny how operational semantics are meant to be consumed by an unfolding, and we are defining ours via folding? Anyway, time to operate the dual construction. The type of traces, being defined coinductively using b, is the biggest object closed under it, i.e. the greatest fixpoint νb. Our denotational semantics are the unfold using the following b-coalgebra, where $\text{Out} : \nu b \to b(\nu b)$ is a destructor observing the very beginning of our trace.

$$\lambda_{\nu b} \circ \Sigma(\text{Out}) : \Sigma(\nu b) \to b(\Sigma(\nu b))$$

So coming up with a distributivity law is always enough to get both semantics for free! Of course, folding using these denotational semantics (or any other algebra, really) is **compositional** (since all folds are compositional by construction). Similarly, unfolding using these operational semantics is **fully abstract** with reference to them (trivially, by definition of full abstraction). The punchline here is that it can be shown that folding a program using these denotational semantics and unfolding it using these operational semantics are actually the same function, that deserves the name of universal semantics. So folding using the denotational semantics, a.k.a. our universal semantics, is always fully abstract, while still being compositional!

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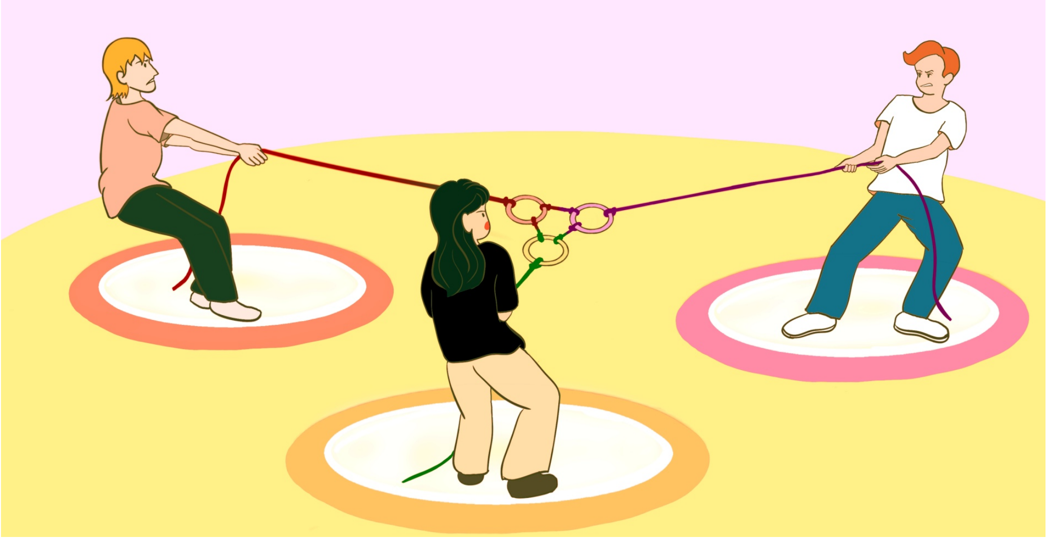

CONSTRUCTING COLIMITS IN A TOPOS USING ELEMENTARY METHODS

ZHAORUI HU | MATHEMATICS

An elementary topos is a category with finite limits, exponentials, and a subobject classifier $t : 1 \to \Omega$. We don't know from the definition that a topos has colimits. Surely we have theorems[1] ensuring that a topos at least has finite colimits, but this is far from intuitive. We might ask for a more straightforward construction of certain colimits in a topos; we want more explicit expressions using limits, exponentials, and the subobject classifier.

Thanks to online posts by Todd Trimble[2], we have an intuitive way of constructing the initial object and coproducts. In the following, we focus on the construction of the initial object as a representative example of a colimit in a topos.

As already suggested in the proof of the Tripleability Theorem (which is used to prove that a topos has finite colimits), colimits such as the initial object can be realized in certain equalizers. Roughly speaking, taking exponentials allows us to get "larger" objects, and taking equalizers allows us to get "smaller" objects. Therefore, working with set-theoretic intuition, we might start by taking a certain equalizer, hoping the result serves as the initial object. However, this approach is too difficult to carry out: even though we have the equalizer and hope it is the initial object, where do we get the unique map from this equalizer to any object in the topos?

Therefore, we need a more systematic approach to solve this problem. The idea is to construct a so-called internal intersection operation, and then take the "intersection of all subsets" to get the desired initial object. We will work with set-theoretic intuition, and, for example, we imagine subobjects as subsets.

More precisely, for any object X, we construct a morphism $PPX \xrightarrow{\cap} PX$. Here, PX is a shorthand for Ω^X, and similarly PPX is a shorthand for Ω^{Ω^X}. Intuitively, we imagine Ω as an object of truth values, and PX is the collection of subsets of X, and PPX is the collection of all sets of subsets of X, and $\bigcap : PPX \to PX$ is like, given any set of subsets of X, we take the intersection of them, and the intersection is a subset of X. Once we have such a morphism, we will be able to take the intersection of all subsets of X, then hopefully this intersection will act as the initial object.

So the issue is how we construct the morphism $\bigcap : PPX \to PX$ with our bare hands. To proceed, we first need to develop some logical symbols within the topos. Why do we need logical symbols? Because they help us

1. Beck's Crude Tripleability Theorem.
2. See the references: the first one gives the construction of basic logical symbols, and the second one includes the constructions of the initial object and coproducts (Trimble 2008; authors 2025).

understand what we are constructing, and they indicate the properties that certain constructions have. Indeed, the only logical symbols we need are $\wedge$, $\Rightarrow$, and $\forall_A$. Luckily, unlike other logical symbols such as $\vee$, these three logical symbols are easy to construct in a topos, meaning that within a few steps of construction using finite limits, exponentials and subobject classifier, we will be able to have them.

The details of constructing them are irrelevant here. We only summarize the important properties they have. Fix an object X. When speaking of subobjects of X, we imagine they are subsets of X. We will use the letters M, N, and K for subobjects of X.

- $\wedge$, or $\cap$: For subobjects M and N, the intersection of M and N is written as $M \cap N$. We have that if $K \leq M$ and $K \leq N$, then $K \leq M \cap N$. As this property suggests, we know that $M \cap N$ can be gotten by taking pullbacks.
- $\Rightarrow$: We have the implication $M \Rightarrow N$ for any M, N. Then for any K, we have that $K \cap M \leq N$ if and only if $K \leq M \Rightarrow N$.
- $\forall_A$:For another object A, we consider the product $X \times A$. We imagine subobjects of $X \times A$ as subsets of the product $X \times A$. So given any such subobject $Z \subseteq X \times A$, intuitively, we can collect those $x \in X$ such that for every $a \in A$, we have $(x, a) \in Z$. We write the resulting subobject of X as $\forall_A.Z$. The property we expect from $\forall_A$ is the following: for any subobject M of X and any subobject Z of $X \times A$, we have $M \times A \leq Z$ if and only if $M \leq \forall_A.Z$.

We still need some internal set-theoretic notions before constructing the map $\bigcap : PPX \to PX$. Indeed, we can reason about the $\in$-relation. For any object X, $\in_X$ is a subobject of $X \times PX$, and again, intuitively, it collects the pairs (x, A), where x is an element of X, A is a subset of X, and $x \in A$.

Now, we have all the ingredients for constructing the map $\bigcap : PPX \to PX$. Intuitively, any element of PPX is a set of subsets of X; we write it as F. Then what is the intersection $\bigcap F$? It is the set of all $x \in X$ satisfying the property that for any subset B of X, if B is in F, then x is in B. Writing this compactly, it is characterized by $\forall B \in PX.(B \in F \Rightarrow x \in B)$. So we imagine that this formula gives us a subset of $PPX \times X$, which are pairs (F, x) that satisfy the property $\forall B \in PX.(B \in F \Rightarrow x \in B)$. Finally, we know that a subobject of $PPX \times X$ corresponds to a map $PPX \to \Omega^X$, i.e., a map $PPX \to PX$, which is the desired internal intersection map.

Although we have omitted most technical details, let us now complete the argument. From some easy observations, we can show that if F and G are subobjects of PX, and if $F \leq G$, then we have $\bigcap G \leq \bigcap F$.

One more important property, which, to my knowledge, is not mentioned in the posts of Todd, is that the composition $PX \xrightarrow{\{\cdot\}} PPX \xrightarrow{\bigcap} PX$ is the same as the identity. Here, the map $\{\cdot\} : PX \to PPX$ is like given any element M in PX, we get an element $\{M\}$ in PPX. The intuition is clear: $\bigcap\{M\} = M$.

Finally, with all the observations above, we know that:

- For any X, write 0_X to be the intersection of "all subsets of X". We can show that 0_X is the smallest subobject of X. Then if M is a subobject of X, then 0_M is isomorphic to 0_X.

- Now, for any object X, we know there is always a monomorphism $1 \to PX$, which picks the object X as a subobject of itself. Then we know 0_1 is isomorphic to 0_{PX}. We also know that 0_X is isomorphic to 0_{PX}, since via the singleton map $\{\cdot\} : X \to PX$, we know X can be viewed as a subobject of PX. We thus get the map $0_1 \to X$.
- To show that maps from 0_1 to any X is unique: take equalizer!

Thus, we have a concrete construction of the initial object. With the internal intersection, one can also construct coproducts concretely. Again, the idea is that we can always take exponentials to go "up", and then take certain intersections, etc., to go "down." However, how to construct coequalizers in this fashion is unclear to me at the moment, and I suspect it will not be as economical as in the case of finite coproducts.

In conclusion, I am satisfied to have found an intuitive and straightforward construction of the initial object in a topos. While the above discussion offers nothing essentially new to the subject, and may even be too limited when it comes to constructing more complicated structures, perhaps the point I want to make is this: although there are powerful theorems guaranteeing the existence of certain structures, it is sometimes enjoyable, and maybe even inspiring, to find a more direct construction, as we did here with the initial object.

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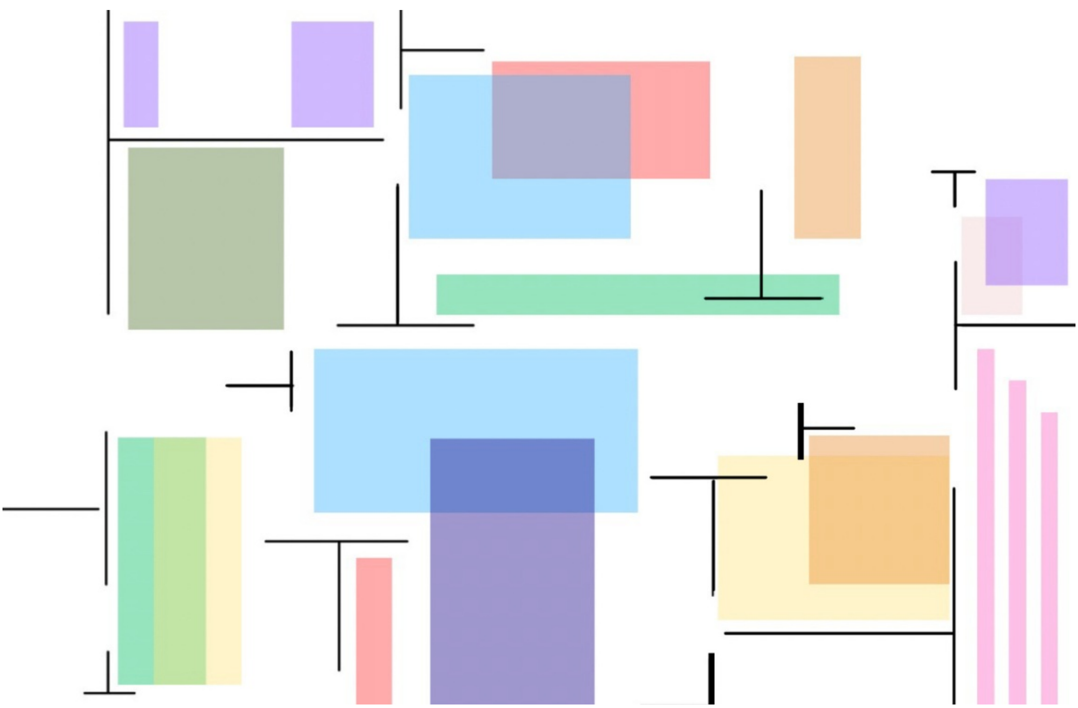

ENGINEERING SEXUAL ORIENTATION

EMMA BATISTONI | LOGIC AND BEYOND

Perhaps Lady Gaga was right - sexual orientation is not our choice: *baby, we were born this way*. This conception has, after all, done much for the mainstream acceptance of certain queer identities. But if we really were simply *born this way*, then ought we only to accept non-cis-heterosexual identities simply because those who identify as such, so to speak, just cannot help it? This seems undesirable, as it leaves open the possibility that there might be something inherently blameworthy about them; such a static conception of sexual orientation moreover does not seem to reflect the lived experiences of many individuals (Diamond 2022, 83, 93).

In addition to that, and perhaps even more pressingly, just what is sexual orientation about to begin with? It is not obvious whether we are referring to sex, gender, or some combination of the two; it is also contentious what these terms really mean, and whether "sex" wasn't already "gender" all along (Butler 2006, 9–10), in the sense that sex categories, which may be thought to be based on cold, hard biological facts, already bear, to a certain extent, the imprint of culturally contingent gender norms.
The "born this way" narrative is at the core of the so-called "folk" conception of sexual orientation, according to which sexual orientation is to be understood as an exclusive preference for a certain sex (but again, what is sex?), and takes priority over sexual preferences - colloquially, "types" - in the sense that types operate within someone's sexual orientation. For instance, someone who is attracted to women and prefers as a general rule tall people will likely not be attracted to a tall man (Halwani 2023, 4–5). This results in three sexual orientations: heterosexuality, homosexuality, and bisexuality.

In my article, I want to provide one main reason why the so-called "folk" conception of sexual orientation is defective and might need to be re-engineered. Before we move on, however, I want to take as axiomatic the following facts: (i) that nobody is entitled to a right to sex, or even a right to be attractive, and (ii) that an individual's (reasonable) sexual preferences are entirely legitimate and to be taken at face value, and the same goes for their self-identification. Any attempt to re-engineer the concept of sexual orientation will need to jostle between on the one hand spiralling into the incel rhetoric of entitlement to sex, because there is no conceivable way to endorse a right to sex and therefore redistribute desire compatibly with a more primitive commitment to basic human rights, and moral authoritarianism on the other, that seeks to dictate what people ought to like and identify as – I view this as incompatible with a commitment to basic autonomy (for a discussion on this topic, see Srinivasan 2021). In other words, any potential blame will not be located at the individual level, barring obviously problematic cases in which preferences are based on explicit objectionable beliefs - for instance, the case of someone justifying their racial sexual preference on explicit racist beliefs. As Andrea Long Chu put it, "nothing good comes of forcing desire to conform to political principle" (Chu 2018)[1].

I focus on what I think is the main defect of the folk conception of sexual orientation: it is not at all clear what it is about. Consider a variation of the famous Ship of Theseus problem: if ♣ self-identifies as lesbian and her partner ♠ starts identifying as a transgender man, and undergoes medical transition, at what point does ♣ stop being a lesbian, if ever? So is sexual orientation to be understood as a preference for sex, gender, or perhaps a combination of both, and what are sex and gender to begin with in this context?

Consider sex first: Halwani attempts to define it as the marks of one's reproductive strategy (Halwani 2023, 3). Under this view, a man's sexual preference for transgender (as in, assigned male at birth) women who retain their penises can be understood as "gynandromorphophilia" (Halwani 2023, 9), because these women[2] have mixed sex markers, i.e. both breasts and penises. Halwani is here making an implicit assumption: that breasts are invariably a marker of the "female" sex, and penises invariably of the "male" sex. I believe this assumption is unjustified, because of mainly two reasons. In the first place, Halwani's assumption threatens to override the lived experiences of the many individuals onto whom the sex binary that he proposes does not neatly apply: what he is saying seems to imply that if a cisgender lesbian is in a relationship with a transgender woman, then she is not really a lesbian after all, and this is incompatible with our commitments. Moreover, Halwani seems to completely overlook the ways in which gender affirming hormonal therapies drastically changes the functioning of sexual organs even in absence of any surgical intervention. But what notion of sex can underlie the concept of sexual orientation, then? At most, and consistently with our commitments, I concede that we may be attracted to traits that are likely by-products of some combination of someone's reproductive strategy, and anatomy, karyotype, and sustained hormonal profile, but this a far cry from Halwani's proposal.

As for gender, consider a self-identified gay man who grounds his, so to speak, gayness, onto his sexual attraction to penises. Would he then be attracted to certain transgender women, and not to many transgender men? Bearing in mind that we do not want to tell people what they ought to be attracted to, nor to override their personal labels, I think this shows that gender cannot ground sexual orientation either, consistently with our commitments.

These brief remarks, I hoped, showed that the folk conception of sexual orientation is defective in, mainly, not being able to accurately reflect the experiences of many queer individuals, and that it is therefore a rather inadequate grounding for, at the very least, the expansion of civil rights. Should we then re-engineer our

1. Here, she was referring in particular to the failed project of political lesbianism.
2. Halwani calls them "biologically male", which I very strongly disagree with.
3. See Chapter 1 of Cull (2024) for a recent coincise overview of conceptual engineering, and Haslanger (2000) for a classical paper thatuses slightly different terminology.

concept of sexual orientation?[3] I believe that these defects point to this.

Lastly, the discussion above may also make one wonder whether we need a concept of sexual orientation at all. As a concluding remark, I want to suggest that such an objection would be somewhat misguided, as I hold that *some* construct is valuable as an analytical tool that enables us to identify patterns in preference that may be socially and politically shaped (see Barn 2022).

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(IL)LOGICAL QUOTES

"F1? Is that with cars?"

"I won't judge, I'm not Italian."

While editing a paper together
"Okay, I change the statement of the theorem and you change the proof."
"Sounds good, what could possibly go wrong?"

"People's opinions are clearly overrated."

"But what if your phone dies, how will you get home?"
"I always know the way home from Science Park"

"I could never date him - he's a platonist!"

"Haskell is only a gateway *drug. Don't stop there. Go sniff some Agda and Idris now."*

"Pi day is my second favourite holiday, after my

OVERHEARD ANY (IL)LOGICAL QUOTES?

SEND THEM IN!

You can submit quotes on our website: resources.illc.uva.nl/TheIllogician/calls

CRYPTIC CLUES

Cryptic clues originated in early 20th century Britain as an evolution of regular crossword puzzles. These crosswords are called *cryptic* for the fact that the clues include some kind of wordplay or hidding meaning, and that the answer usually does not match the plain reading of the clue. This (often more difficult) variety of crosswords gained popularity in the UK and beyond, with cryptic crosswords appearing regularly in major newspapers like *The Times* and *The Guardian*.

In their modern format, cryptic clues generally include the following:

1. A *definition*: Found at the beginning or end of the clue, the plain reading of this part by itself will describe the answer to the puzzle, just as a clue in a regular crossword. However, this description will often only match the answer in a broader, humorous, or more roundabout way and hence the definition itself is often not enough to solve the puzzle.
2. *Wordplay*: The other part of the clue will form some sort of wordplay that, when solved, also yields the answer to the puzzle. The wordplay itself can usually itself be divided into wordplay indicator, words that hint at what kind of wordplay needs to be applied, and fodder, i.e. words on which the wordplay acts.

EXAMPLE

Single tutors accidentally fish in freshwater. (5)

Here, the wordplay part of the clue is *single tutors accidentally*. The first indicator, *single*, acts on the fodder, *tutors*, and tells us to take not the plural but the singular form of *tutors*, i.e. *tutor*. Secondly, *accidentally* is an anagram inidicator, telling us to rearrange the letters of *tutor* to find the answer. (Imagine the letters getting into an accident and being disordered as a result.) Doing so gives us *trout*, an answer that matches the definition part of the clue, *fish in freshwater*.

CLUES

In the following you find three logic themed cryptic clues. For hints and solutions visit resources.illc.uva.nl/TheIllogician/posts/2025-ii-solutions.

TWICE NEGATING DIAMOND SHIFTS A BOX (5)

☐☐☐☐☐

ARTICLE BACK STORY CONTAINS MANY SENTENCES (6)

☐☐☐☐☐☐

CRAZY CARDINAL INHALES NITROGEN, TARGETS PAPACY (8)

☐☐☐☐☐☐☐☐

ACADEMIC DESTINY GENERATOR

WHAT RESEARCH FIELD WERE YOU BORN TO EXPLORE?*

DAY OF BIRTH RANGE	BIRTH MONTH	INITIAL OF FIRST NAME
1-3: FORMAL	JANUARY: ALGEBRAIC	A-B: MODEL THEORY
4–6: QUANTUM	FEBRUARY : HYPERINTENSIONAL	C-D: CRYPTOGRAPHY
7–9 : PARACONSISTENT	MARCH: INTUITIONISTIC	E-F: MEREOLOGY
10–12: COMPUTATIONAL	APRIL: DEONTIC	G-H: EPISTEMOLOGY
13–15: FINITE	MAY: MODAL	I-J: TOPOLOGY
16–18: CATEGORICAL	JUNE: HIGHER-ORDER	K-L: LOGIC
19–21: BILATERAL	JULY: PROBABILISTIC	M-N: PROOF THEORY
22–23: MANY-VALUED	AUGUST: DYNAMIC	O-P: SEMANTICS
24–25: NON-WELLFOUNDED	SEPTEMBER: REVERSE	Q-R: GAME THEORY
26–27: ALGORITHMIC	OCTOBER: INQUISITIVE	S-T: TYPE THEORY
28–29: NON-STANDARD	NOVEMBER: DESCRIPTIVE	U-V: SET THEORY
30–31: FIXED-POINT	DECEMBER: INFINITARY	W-Z: COMPLEXITY

*If it actually matches your research interest, you will get a free coffee at Nikhef!