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The Relevance of Judgment for Philosophy of Science

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Interfacing logic and judgment

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1 Introduction

Good judgment is an intellectual quality that goes far beyond being well-informed and possessing good reasoning skills in drawing conclusions from available evidence. Beyond these basic competences, arriving at a judgment involves a further exercise of insight in a space of free decisions. This surplus may suggest a tension with logic, a discipline often seen as imposing automatic rules whose conclusions are 'forced' upon us, a telling, if not ominous, terminology. But the reality is surely cooperation. Logic is an indispensable *pre-processing* tool: good judgments should be based on good evidence and good inferences. And the role of logic does not stop there: it is also a *post-processing* tool for critical inspection: judgments with absurd logical consequences, no matter how profound, are in trouble and should be modified or abandoned.

Yet these obvious connections between judgment and logic are not what this brief paper is about. I rather want to stress the importance of judgment calls *in the practice of logic* itself. My examples will be mainly 'small judgments' about what to infer logically from data at our disposal, or how to make sense of what we see or hear. These are things we do all through life. More momentous 'large judgments' arise with accepting or rejecting a complete conceptual framework or scientific theory, reflecting the large fateful decisions we make only now and then in the course of our lives. For what it is worth, I believe that large and small decisions of this sort lie on an intellectual continuum, but I leave it to the reader to judge the reach of the analysis to follow.

2 Logic: structure and agency

Two faces of logic. A brief explanation may be in order of what is meant by 'logic' in what follows. In one sense logic is the study of a world of abstract relations between propositions and formal systems, and many other structures that do not involve actual reasoners. This is the foundational face of the field. But at the same time, logic is used by agents, or at least we can use it to model real and ideal agents, and this is a second face of the field,

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historically entangled with the first. What follows will be in the second, dynamic mode: judgment arises with 'logic in action' as used by agents.

Judgments as acts. The term "judgment" is used routinely by logicians for an *act* of asserting a proposition, or the *product* of such an act. The difference is often blurred (as it is in our daily natural language), though Frege did indeed make a clear distinction between content and assertion. Be this as it may, the dynamics is obvious: logical reasoning involves acts of drawing consequences, selecting and arranging premises, choosing pathways toward conclusions, and much else.[1] What is more, in addition to more narrowly reasoning-related acts, there are other basic ones.

Information-producing acts. Reasoning is an informational process that lives inside a larger realm of questions, tasks, and inquiry. Most sciences, and for that matter: daily life, crucially involve a mixture of inference with *observation* and *communication*. This point was already noted by the Mohist logicians in China in the 5$^{\text{th}}$ century BC. Their elegant compact statement was "*zhi wen shuo qin*": 'knowledge arises from hearing, demonstration, and experience'. In modern terms, information flows because of what we observe, what is communicated to us, plus the inferences that accompany these, Zhang & Liu 2007.[2] This interplay of many informational actions is not just ancient philosophy, but the reality of everyday agency and the actual functioning of science. Moreover, broader information-producing acts interact naturally with inferences in a stricter sense, and in doing so, they still fall within the scope of logic: they obey laws that are investigated in contemporary dynamic logics of information, cf. van Benthem 2011. This broader dynamic perspective on what happens in information-driven agency is the setting in which we will explore the role of judgment in logic.

3 Judgment and inference

What to infer. A particular valid inference from given premises leaves no room for judgment when passing to the conclusion. This step is automatic, and can in fact be automated on machines. But a much wider open space arises when we consider *which inferences* to draw from given premises. This asks for a guiding question or a *purpose*, and judgment is crucial here. Sherlock Holmes makes the essential relevant inferences from the observed facts, and in this, he has better judgment than most of us. An interesting broad cooperative perspective is explored in the dissertation Icard 2014: semantics supplies the "what" of valid inference, proof theory adds the

[1] Some authors say that reasoning in fact proceeds from judgment act to judgment act, but this may not do justice to the variety of logical acts and epistemic attitudes that agents actually engage in.

[2] A Mohist example is this. I see an object in a dark room and an object outside that is white, someone tells me that the objects have the same color. I now infer that the object in the dark room is white.

"how" of wielding this repertoire, but equally important is the "why" of drawing which conclusion, which is always goal-oriented and involves links with decision theory.[3]

Judgment in mathematical proof. Going beyond single inferences, mathematical proofs encode sustained reasoning through *complex structure* at a higher aggregation level of successive inferences arranged in patterns. While checking for correctness of each step is logic in automatic mode, the complex structure brings issues of its own. On a simplistic view, proofs establish one conclusion: their last statement, and we just have to check each link in the chain or proof tree. But in addition to a final conclusion (and of course also intermediate ones), good proofs have a *moral*: a general point of the proof providing reasons and further pathways whose identification and communication is a matter of judgment going far beyond checking for correctness. Of course, judgments of proof morals can be a matter of continuing debate, witness the century of thinking about the essence of Cantor's diagonal argument, Yanovsky 2003.

Choice of a logical system. But even behind fixed single inference patterns, larger judgments loom in our earlier sense. Is the conclusion of a reductio ad absurdum automatically forced upon us? There is an a priori decision here: classical logic will tell us to accept, intuitionistic logic accepts one version and rejects another. Choices *between different logical systems* require judgment (though there is also dogmatic slumbering in the profession), and in these judgments, a wide range of considerations plays a role, from observable facts about human reasoning to methodological virtues of general architecture, just as in choosing between scientific theories. Of course, these are large judgments by theorists, not by day to day users of logic, but even the latter have the ability in principle to reflect on their reasoning practices.

Identifying the relevant pattern. So far, as is common in the philosophy of logic, we considered abstract patterns placed before us. But in practice, agents need to *discern the relevant patterns* first in the concrete information format at their disposal. Here is a telling example from Mercier et al. 2017: "Twenty farmers own at most fifteen cows each. So: At least two farmers own the same number of cows." Is this conclusion correct? And if so, why? One might be inclined to look for some quantificational pattern close to first-order logic, but that would not work and miss the essence. What is arguably at work here is the mathematical *Pigeon Hole Principle*. Putting 20 objects into the 16 boxes of 'owning i cows' with $0 \leq i \leq 15$ must place at least

[3]Icard provides computational models for investigating the decision-theoretic "why" of inference in precise terms.

two objects in the same box.[4] More generally, discerning logical patterns in real reasoning couched in language or other vehicles for information (visual patterns, movements) requires sometimes quite non-trivial judgments going beyond purely formal skills.

Summary. Reasoning is replete with judgment calls in deciding what to infer, in determining what a complex proof means, or in adopting a logical system. In addition, judgment is essential in bringing logic to bear on concrete reasoning scenarios.

4 Judgment and representation dynamics

Representation and information update. A basic aspect of the broader information processes that we highlighted in the above is their need for *representation* of the relevant facts. These representations can be purely linguistic, but they do not have to be. "A waiter in a café takes orders from three people for wine, beer, and water. Another waiter returns carrying three glasses. What will unfold is this. The new waiter asks who has, say, the wine, puts that, asks who has the beer, puts that, and then puts the remaining glass without further ado." The information flow here involves two questions, two answers, and one final inference by the waiter. As the scenario unfolds, information gets updated as directed by the questions.

One can describe this process in terms of sentences in the language of the participants, but more abstract representations also make sense. Initially, there are 6 options for assigning the glasses that we can diagram using combinations WBw, WwB, etc. After the first question has been answered, the diagram simplifies to 2 possibilities: Bw and wB. The second question and answer reduce this to one, just the actual state of affairs, and the third glass can be put based on a simple (probably unconscious) inference. This is just a simple example, and many other types of representation occur across a spectrum from detailed syntactic language forms to abstract diagrams. Based on this example plus a wealth of literature that we cannot survey here, we make a sweeping claim. Much of logical semantics in philosophy or linguistics is about creating models that can serve as representations of relevant information for agents.

Update rules for representations. As we saw, *updates change models* representing relevant information for a task at hand. There are many rules in logic that describe such changes, from elimination of possibilities as above to Bayesian conditioning or other tools in formal epistemology. In particular, incoming information can be taken in different ways, from 'hard', indubitable and irrevocable, to 'soft', keeping some reservations and fallbacks around. Such varieties may need richer representations.

[4] Van Benthem & Icard 2023 explore the complex entanglement of logic and elementary mathematics.

For instance, *plausibility models*, van Benthem 2011, order epistemic possibilities s, t by a binary order "s is at least as plausible as t" from the standpoint of an agent. One can think of the set of possibilities as what is still in play for the agent after all others have been ruled out in the course of inquiry, while the plausibility order records current expectations or beliefs about these various possibilities, usually based on a past history of evidence gathering. Now a hard update with the proposition φ will totally eliminate all current possibilities satisfying $\neg\varphi$, while a soft update might keep all possibilities around, but put the ones satisfying φ on top as most plausible or most relevant in the updated ordering of the options. Clearly, these are just two out of many options for letting new information impact one's current epistemic-doxastic state, and dynamic epistemic and doxastic logics study a wide range of options.

Uptake requires judgment. The variety of possible update rules for information is often seen as an embarrassment for a research area, being a way of "riding off madly in all four directions", Leacock 1911. But we can also see it as a rich repertoire of options available to intelligent agents who must make a judgment on the quality of new information received and then respond accordingly. Well-considered *uptake* is crucial to communication, and it requires judgments of content, of the communicative scenario or interactive game one is in, of the reliability of sources, and much more.

Balance of logic and free judgment. The preceding perspective raises a perhaps somewhat technical question of independent interest. Is a large and growing variety of update rules the best way of describing intelligent agency, or could there be one logical *Mother Rule* for information update which is inescapable, but whose design makes uptake a free choice parameter? Proposals to this effect include the dynamic-epistemic product update rules for knowledge, Baltag, Moss & Solecki 1998, and for beliefs in Baltag & Smets 2006. By locating some logically inescapable grand pattern for update, this design in fact highlights the precise locus where additional judgment is needed in setting the parameter, providing one more instance of logic and judgment supporting each other.

The open space of representations. Update rules and representation structures from formal semantics, once mathematically defined, follow one particular linguistic groove. But we can also step outside of such a model during the process of inquiry, and radically *change the very representation format,* perhaps switching the whole language employed until now. Movements in this open space of representations occur all the time in logical research, and they require larger judgments of the sort we mentioned before: about the best choice of a new conceptual framework.

Summary. Representation is essential to inquiry. Judgment is required in choosing or changing representations and in deciding on the uptake of

new information in a given representation. Logic can describe this process in a way that identifies the choice points in representation formats and their updates where judgment is needed.

Coda: the dynamic boundary of logic and judgment. Here is one last crucial aspect of our topic as we see it here. Decisions as to the division of labor between logical rules and free judgments need not be taken once and for all. In reality, the interface of automated rules vs. free decisions of agents can shift. We may decide to *trade in some judgmental space* for automated rules when this is practical (like in AI support systems), in analogy with how our brains can automate cognitive tasks performed consciously at first.[5]

5 Judgment in formal epistemology

Many points made in the preceding sections about logic and judgment could be made equally well in the broader area of formal epistemology, Arlo-Costa, van Benthem & Hendricks 2017. In this interface area, logic meets with formal philosophy and other fields, and many standard themes involve judgment calls. We confine ourselves to just a few points going beyond those discussed in earlier sections.

Knowledge claims and belief formation. Formal epistemology contains many accounts of what it means to know, or be entitled to making a knowledge claim, from Plato's justified true belief to modern relevant-alternatives or belief-stability theories. In a longer version of this paper, I would argue that belief formation by agents on the basis of the available evidence leaves essential room for judgment, but for here, I just highlight one interesting feature of beliefs that seems of general relevance to me.

The right to be wrong and revising judgments. Our emphasis on logic in preceding sections may have suggested that judgments are needed on the way to truth and knowledge, with a sort of continuous march toward *correctness*. But this is not the whole story. Beliefs can legitimately *change* under the pressure of new evidence, and it has even been said that a major sign of intelligence is not always being right, but a talent for creative *correction*, Popper 1962. Belief changes can be described in belief revision theory, Gaerdenfors 1986, formal learning theory, Kelly 1996, or in the earlier dynamic-epistemic logics, in the small by changing plausibility orders,

[5]Such automated rules in intellectual performance need not come from logic alone. The probabilistic architecture of Bod, Scha & Sima'an 2003 shows how consciously performed linguistic and logical tasks can over time become a matter of pattern recognition in memory.

But in an opposite direction, we can just as well *take back our freedom*, and open up a rule system we had submitted to for conscious judgment calls. These are not just speculations. For a live illustration, look at current discussions of video-assisted refereeing in professional soccer, or if this is too down to earth, think of the role of judgment versus rules and automation in the Law, Ulenaers 2020.

and in the large by changing representation formats. But even beyond specific frameworks, logic can help release the creativity needed in revision by providing refutations of current beliefs or positions, not just through new evidence, but also in argument or debate. The general moral is that judgments need never be final: they can be shown wrong, and just as in the Law, appeal and revision must always be possible.

Judgment and further epistemic talents. To keep this dynamic of revision alive, a broader canvas may be needed than what has been discussed in this paper. Judgment is a talent or a competence, but equally important are its epistemic siblings: a talent for *doubt*, and indeed: for knowing when to *withhold* or *retract judgment*. This is true in epistemology, but the same talents are valuable in the practice and theory of logic.

6 Conclusion

In this very brief and frankly rather apodictic piece, I have claimed that logical skills and a talent for judgment are natural companions. Looking at the practice of logic itself, both by its theorists and its users, agents need judgment all the time in logical tasks, from drawing the right inferences to choosing representations and modes of information update. In all this, logical analysis can even help identify where logic ends and judgment must take over, though the border between free space for judgment and logical rules can also shift dynamically.

In arguing for this tandem, my examples were mainly about small judgments by agents engaged in given tasks, but larger judgments concerning format or logical system also appeared occasionally. I do not believe that there is a quantum jump here, and much of what I said should hold in the large as much as in the small. But that claim needs more evidence than I can present here.

Finally, a disclaimer may be in place. With all the above, I have not claimed that judgment is exclusively tied to logical reasoning. General argumentation and further considerations play a role, as well as the ability to see new perspectives. Indeed, insisting on too much precision with judgment calls can be tedious: recall Aristotle's famous dictum that it is the hallmark of an educated mind to give each subject the amount of precision it warrants, Aristotle 2004—said by, perhaps not accidentally, a creative logician. But I do hope to have suggested that, whatever other virtues it may have, logic provides an interesting lens for spotting and enhancing judgment calls.

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