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Models and Representations in Science

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What logic represents

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Abstract. In this short note, we discuss a few senses in which logic represents natural language and natural reasoning and then fan out to a broader perspective on applied logical analysis.

1 Introduction

The discipline of logic started when thinkers in Antiquity noticed recurrent patterns in valid and invalid inferences occurring in reasoning practices and found that these could be studied as such. Since reasoning usually takes place couched in natural language, a medium whose syntax serves many further functions, these reasoning patterns were made explicit using special notations for logical forms. In modern logic, logical forms live in formal languages with a complete syntax and semantics that start looking like full-fledged alternatives to natural language, a line taken in the famous 'equality in principle' thesis for natural and formal languages in Montague 1974. This raises the issue of what logical systems model or represent, and we will phrase the following discussion in these terms, though we will also question the full-language methodology in the end. Our light discussion will gradually draw in the choice of semantic structures and other basic themes in the design of logical systems.

2 Logical syntax and representing natural language

Patterns. The emphasis on discovery of *patterns* is a common theme in studies of the historical origins of logic and other academic disciplines, cf. Bod 2022. And one might think that these patterns are linguistic, since they were presumably extracted from natural language, our common medium of expression and communication. But how should we think of the matching of natural language 'in the wild' with designed logical patterns?

Syntax. Consider the common didactic practice of training students in 'translations' of inferences stated in natural language in some formalism such as first-order predicate logic. We can think of this as projecting natural language into a simpler language of forms that highlight just the structure that is relevant to inferences. The Latin diminutive 'formula' is particularly apt here in its literal meaning of 'little form', as we are aiming for simplicity.

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Grammar and simplicity. Yet, what simplicity means is a vexed issue, and at the level of syntax alone, not that much comes to mind. One might compare the length or other measures of *syntactic complexity* of natural language sentences with that of their formula translations, though I am not aware of significant results in this direction. Indeed, logical syntax can even add space-consuming devices that do not occur overtly in natural language expressions, such as scope indications and variable binding. On the positive side one could see these extras as logical syntax modeling not just expressions, but broader linguistic mechanisms. Variable binding offers a simple model for anaphora and discourse coherence, and thus its more intricate structure beyond natural language surface syntax comes with its own benefits.

Another angle on syntactic simplicity places the focus on *grammatical complexity*. Grammars for formal languages are usually context-free: cf. van Benthem 1988, a study of logical syntax, for details—and exceptions. In contrast, grammars for natural languages are often context-sensitive, that is, higher up in the grammatical Chomsky Hierarchy of complexity.

But arguably, the simplicity and utility of logical formulas rather has to do with their *uses*, namely, their function in analyzing or recognizing inferences. We now turn to this angle.

Proof systems and text grammar. In the above reconstruction of historical origins, the point of logical formulas is their role in making the structure of inferences explicit. Formal notations such as, say, "A or B, not-$A \Rightarrow B$" do just this, with sentential variables A, B for parts of a possibly much more complex linguistic expression whose precise nature does not matter, and a focus on the specific logical expressions of disjunction and negation that do matter to the validity of the given inference, cf. Bolzano 2014, Bonnay & Westerståhl 2016.

In doing so, we extend the above view of sentence grammar to one of *text grammar*: the logical structure of sequences of sentences involved in an inference. When chaining individual inferences into more complex proofs, this text grammar also brings its own technical notions that may go beyond natural language texts, such as long-distance dependency management of conclusions on assumptions in natural deduction, cf. Prawitz 1965.

Variety. One striking feature of the logical study of proofs is the *variety of available systems*, from Hilbert-style axiomatic to many styles of natural deduction. One can think of these as different computational implementations of deduction for practical purposes, but one can also take them more realistically. They are then proposals for representing a particular style of reasoning, a claim suggested by the terminology 'natural deduction'. I am inclined to the latter view, but the criterion of success for such claims is not always clear, since logical proof systems are also meant as a tool to be learnt, and *improve* a given reasoning practice. Thus they exhibit the same two faces as the discipline of logic itself, harboring both descriptive and normative aspects in its ambitions, a tension that I will mostly ignore here.

Digression: natural logic. The idea that formal languages are indispensable for analyzing inference in natural language has not gone unchallenged. The program of *'natural logic'*, van Benthem 2008, Moss 2015, uses natural language syntax as is to represent some inference practices in ways that are simpler than the usual logical representations. The issue then shifts to *when* it becomes more profitable to make the transition to formal logical modeling.

Meanings: inferential and semantic. The discussion so far may suffice for an inferentialist who holds that proof rules determine the meanings of logical expressions. But I myself think semantically and want an independent analysis of meanings, if only, to judge whether a proposed proof system makes sense. What follows will be a semantic perspective bringing to light further plurality of representation in logic. Even so, many of the following themes may also make sense in a purely proof-theoretic treatment which I do not pursue here.

3 Logical semantics and representing natural language

The usual translation exercises in logic courses do not seem to be purely syntactic, as the first-order language used comes with an intended semantics that involves two features.

Semantics 1: Conceptual frameworks. One component of a semantics is a structured view of what the described reality looks like. In a common view of predicate logic, these are models with a domain of individual objects and predicates and functions over these. This can be seen as a proposal for a *conceptual framework* for natural language and inference. And as such, it is a choice of representation since natural language does not force us to think in just this way. In fact, some philosophers and linguists have rejected an ontology with individual objects as primary citizens, cf. Keenan & Faltz 1986. And even for formal languages, logic itself has an alternative in the long-standing algebraic tradition, Sanchez Valencia 2004, that works with, one might say, domains of concepts with various interrelations, and only admits underlying objects for the algebras if these can be introduced through representation theorems.

Variety is the rule in logic. The preceding is not a criticism of the standard Tarski semantics for predicate logic. The latter has proved a widely useful representation for human inference, for automating reasoning, and for proving results stating deep insights into the metatheory of reasoning with predicates and quantifiers. The point is just that, as with any proposed representation, there can be attractive alternatives. There are many further examples of such framework options in logic, especially when we turn to modal expressions that go beyond the static here and now. For instance, many formats exist for representing the pervasive temporal reasoning in natural language: tense logic with Past and Future operators, interpreted on points or alternatively on intervals, but also different logical forms provided by a two-sorted predicate logic over points in time, or yet other structures, cf. van Benthem 1995.

Semantics 2: Mechanisms of interpretation. The second fundamental aspect of a semantics is how it makes the connection between the syntax of a language and the intended models. Famously, for predicate logic, this *interpretation mechanism* is compositional and based on Tarski's notion of satisfaction which involves assignments of objects to variables as 'states' of the interpretation process. And yet again, there are alternatives to such a package of type of models plus type of interpretation mechanism. For instance, dynamic semantics, cf. the survey Nouwen, Brasoveanu, van Eijck & Visser 2016, has an alternative more procedural view of what happens when we interpret expressions with anaphora, and there are yet other attractive formats, such as discourse representation theory, Kamp & Reyle 1993, or game-theoretic semantics, Hintikka & Sandu 1997. What all these examples make abundantly clear is that logic can also model many different views of the semantic interpretation process.

Compositionality. These options also illustrate another virtue of logical modeling. A general analysis and design principle for semantic interpretation of all the above kinds is *compositionality*, Baltag, van Benthem & Westerståhl 2023. Logical languages are both the origin and the most perspicuous illustration of how this methodology works. Moreover, it is their abstract simplicity that helps us develop a range of compositional interpretation procedures.

4 Task dependent representation: Functions of natural language

Our discussion so far has left out an important parameter. Representation is usually there for some *purpose*, and its adequacy can depend on that purpose. Now natural language has many different functions, and so far we have only encountered two of these.

World description. Natural language is a medium for *describing* what the world is like, or what the language users take the world to be like. Predicate logic offers a model for that function: its language represents the structure of natural language sentences describing situations in the world, while its models are a way of representing those situations.

Theoretical terms. The simple term 'description' quickly gets more complex when we move away from simple situations in the world and look at the many theoretical terms in language. When we call a person "friendly", we do not assign an observable property, but express a complex expectation about behavior of that person over time. And explicit modal expressions like "believe" even populate the physical world with 'constructs': unobservable mental attitudes that serve as postulated theoretical terms to make sense of human behavior, van Benthem 1983, just as physicists postulate forces or fields in their theories to make sense of observable reality. Thus, the conceptual framework of a semantics may also include quite complex abstract notions that shape our perspective of, and expectations about the world.

Inference. However, this note started with another function of natural language, namely, as a vehicle for *inferences.* It is not obvious that the same logical forms that help analyze inference are also optimal for describing the world. Already van Benthem 1987 asked why it is that the same representation in formulas of predicate logic works so well for such different purposes. Even so, divergences do exist in logic, for instance, with the use of Skolem forms in resolution theorem proving, which are not easily humanly interpretable, Robinson 1965.

Communication and coordination. A third crucial function of natural language that has attracted attention from logicians is *communication.* Again it is not obvious why the logical forms that serve description or inference would also serve this further purpose. And indeed, current dynamic-epistemic logics for communication employ additional logical operators for information updates that have no direct counterparts in natural language, Baltag & Renne 2016. We will not discuss the representational role of the latter logical forms here.

Beyond information. Communication is a way of *coordinating behavior* which involves more than the informational focus of logical modeling. Successful communication is at the same time a way of agenda management, achieving goal alignment, and even of achieving the emotional resonance that is crucial to understanding, learning, and shared agency.

5 Connecting representations to what they represent

From natural to formal languages. What is the connection between natural language and logical formalisms designed for functions like those discussed above? One might think that this is just an art of modeling based on experience, but sometimes more can be said.

Translation. Logic texts often use the term *'translation'* from natural into formal languages, but this may suggest too much. A predicate-logical formula is seldom a faithful rendering of a natural language sentence, except for the simple type of discourse of the "Mary knows John" type one finds, for instance, in factual data bases, or in simple natural language processing.

Paraphrase. For many other purposes, one can view a logical formula as a *paraphrase* of a natural language sentence, geared toward representing the essentials needed for a particular task. Examples of this abound in work on 'logical AI', McCarthy 2001, where the logical text describing the relevant content of a problem to be solved may diverge considerably from what the natural language version says, both qua formula structure and qua arrangement of the text. This paraphrasing ability is much more widespread and useful than translation skills, and while it cannot be made algorithmic like translation, it can be trained and honed.

Maintaining harmony. One can also juxtapose natural language and representing logical formulas without any claim of intrinsic adequacy except for demanding that actions in the two realms should stay in step. The latter view is made more precise in the analysis of logical modeling in Moss & Westerståhl 2023. The authors assume that natural language sentences describe 'situations' or 'scenarios', seen as parts of the world, or as empirically real mental pictures that we form of the world. We are then free to connect sentences with formulas and situations with models in any way we like, but the criterion of adequacy is the harmony expressed in the following diagram, whose arrow structure should commute:

$$\begin{tikzcd}
\text{Situations} \arrow[r, "\text{represent}"] & \text{Models} \\
\text{Sentences} \arrow[u, "\text{description}"'] \arrow[r, "\text{paraphrase}"] & \text{Formulas} \arrow[u, "\text{truth definition}"']
\end{tikzcd}$$

Starting with a sentence or text, if we first move to logical formulas at the bottom and then, staying in the logical realm, follow the formal truth definition in logical models upward, we should get the same effect as first following the informal description upward in the empirical realm, and only then follow the representation into logical models. Here the arrows can also stand for relations rather than functions, with 'truth in a model' as an obvious example. This leaves much more freedom for how the logical theorist decides to make connections.

Note that a commuting diagram for just one concrete sentence and formula can be constructed entirely ad-hoc. To make the analysis do real work, we will want to demand commutation for all representation pairs in some family to be specified in the intended application.

Similar attunement diagrams make sense for inference, where logical inferences between formulas should track actual reasoning steps in natural language. And they can also be used to check whether proposed formal update mechanisms track real information flow.

General tracking. Now this perspective might be considered 'behaviorist' since we do not apply any criterion of intrinsic resemblance, but only demand that representations stay attuned to the empirical practice they are modeling. But this generality is also a virtue. Indeed, the preceding methodology, which can be made precise in a general category-theoretic setting, applies to a wide range of forms of 'tracking' one system with the aid of another.

Connecting situations and models. The preceding diagram raises a further question: what connects real structures like situations or scenarios with the models in a logical semantics? There are many candidates in the literature, from isomorphism to weaker simulations or embeddings. We will discuss these later under the heading of transformations and invariants. But as with the above paraphrasing, assigning formal models to real-world scenarios may be something of an art based on experience, rather than an algorithmic procedure.

6 Equivalences within the logical language

Next we move to representation *inside logic itself*, as there are also substantial issues here.

From formulas to propositions. Once inside the realm of logic, perhaps the major theme for a logical system is exploring the *valid laws* governing reasoning in the domain under study. This adds a complication to our earlier discussion. We may have suggested so far that the representing object for a sentence is literally the logical formula associated with it, in any of the manners outlined in the preceding section. However, the valid laws of a logical system induce a notion of *equivalence* between formulas, and thus, the real logical object doing the representing is the structure underlying that equivalence class. In common philosophical parlance, we are after the *proposition* expressed by these equivalent formulas.

Alternative logics. This perspective raises deep issues that run through the logical literature. Which notion of equivalence is appropriate to the external domain being represented? For instance, does a sentence of the form not-not-S express the same proposition as sentence S? The answer is positive in a classical truth-oriented perspective, but negative when representing a constructive mathematical practice where negation means refutation. One can construe logical proposals for dealing with constructivity, or in recent years: hyper-intensionality, as offering different, less or more fine-grained, views of what one takes to be the relevant structure of propositions. These alternatives may arise with different choices of semantic frameworks, when e.g., truth gets replaced in favor of 'support', or they could have more proof-theoretic motivations, as is the case for various constructive logics.

A landscape of levels. The resulting variety forms a landscape where different systems focus on *different 'levels of grain'* in representing their object of study. This variety can even arise when we fix one particular formal language, say the standard one of propositional logic. At one extreme one can find coarse representations of propositions as sets of worlds, at another, the very syntax of the logical formulas themselves. This landscape is not linearly ordered, and has many gradients: sets versus topologies, qualitative versus quantitative, and so on.

The above is just a very brief summary of some well-known facts about logic. I stated them merely to emphasize their relevance to how we can represent empirical practices, and as background for the following points that are sometimes neglected.

Freedom in language design. A multi-level landscape of options is entangled with the *design of logical languages.* More fine-grained representing structures can interpret richer logical languages, so the issue should not just be strength of identification but also the medium of representation. Some literature on alternative logics ignores this point, focusing on one standard language, say, of propositional logic, without raising the question whether this formal language is the best medium for the semantic picture one is advocating.

To me there is a serious conceptual desideratum of *expressive harmony* between a semantic framework and the logical language one chooses to access it with. And this harmony is also important technically: infelicitous language design can make proof systems for validities opaque and completeness extremely hard to prove, merely by self-imposed restrictions.

Co-existence instead of competition: translations. A next point to note is that there need not be a 'right' representation for a given reasoning practice in this landscape of options Many languages and models may make sense, and what logic then offers is a total systematic picture. But there is a force for coherence in logic, in terms of a web of systematic *translations* and other forms of correlation that run across and facilitate commensurability for many logical systems.

No preferred direction. Next, there is *no preferred direction* for design in the landscape. In the history of science, coarsenings have proven just as fruitful as refinements. Compare the coarser qualitative perspective of Topology with the detailed quantitative level of Analysis. Coarser levels can highlight essentials that were invisible down below, such as the simplicity of continuous maps in Topology versus the epsilon-delta definitions in Analysis. For a more philosophical example, a 'hyper-intensional' logic is not automatically better than a 'standard modal' one: each may offer insights at its own level. I myself would even say that, if hyper-intensional logics had come first in history, there would have been a later major discovery that it also makes sense to throw away detail and introduce the standard modal logics.

Syntax from 'what' to 'how'. My third and final point concerns *pure syntax*, usually seen as a non-contender for the structure of propositions. However this may be, syntax represents something essential, even when logical equivalence identifies different formulas. Despite such validities, say, "not-(P or Q)" and "not-P and not-Q" are *different ways of getting to* the same denoted proposition. This 'how' can be seen clearly in the evaluation games associated with different formulas in game-theoretic semantics, and the various strategies that players have in these, which give different reasons for the truth of a formula in a given model. This combination of 'how' and 'what' seems crucial to our use of language.

Aside: computing. Syntax is also essential to *computation*, which needs code. While the algebraic terms $x + x^2$ and $x^2 + x$ always denote the same number, they correspond to different procedures for producing that number, and in general some procedures can be more perspicuous or efficient than others. It has even been suggested that such algorithmic differences are crucial to an algorithmic understanding of the Fregean notion of 'sense', Moschovakis 1993.

7 Invariances among logical models

Having discussed equivalence in logical syntax, let us now turn to equivalence in logical semantics. Formal models as they stand are seldom the true structures one has in mind. Models usually come with *invariance relations* that leave 'the same structure' intact. The standard example is isomorphism, which says that it is immaterial to the structure which actual objects do the representing. Accordingly, logical formulas will be true for objects in one model iff they are true for the images of those objects under an isomorphic map to another model. This requirement is even part of the definition of a logic in Abstract Model Theory.

Invariance relations. But as with logical equivalence, there are many options for invariance relations between models, depending on what underlying structure is the focus of interest. Isomorphism is a very fine sieve, but, say, in modal logic, a much coarser identification is often taken to represent the crucial accessibility structure of patterns of possible worlds or process states, namely, some form of bisimulation, Blackburn, de Rijke & Venema 2001.

Erlanger Program. This variety matches practice in mathematics, in a tradition going back to the Erlanger Program, Klein 1872. A mathematical theory describes structures that come with a *group of designated transformations* that define when two different manifestations of that structure are 'the same' from the perspective of the theory. For instance, Euclidean Geometry looks at spaces under the standard transformations of translation, rotation and reflection, while Topology identifies spatial structures under the much coarser notion of homeomorphism. Both perspectives have their uses, neither is 'better' than the other.

Systematic attention to the role of transformations and invariance is less common in the philosophical or linguistic literature. But it does occur implicitly when we realize that an ontology needs to come with a 'criterion of identity' between objects, Noonan & Curtis 2022, or perhaps better: between different semantic ways of getting to the same structure.

8 Connecting logical languages and logical models

Invariants and the emergence of language. The two main perspectives in the preceding discussion, formal language and semantic models, are intimately connected. Given any notion of transformation between structures, there will be *invariants*: predicates whose truth is not affected by moving from objects in one model to those in another model related by a transformation. Already Helmholtz 1883 saw such invariances as crucial to *the genesis of languages*, since languages will tend to express patterns that we reuse in different manifestations of the same empirical situation or scenario. This theme underlies the above Erlanger Program and the ubiquity of invariance in physics and other disciplines, Suppes 2003. In philosophy, the theme also occurs occasionally, as in the situation theory of Barwise & Perry 1983 with its emphasis on the informational constraints that structure our world.

Logical languages and invariance. The harmony of semantic invariance relations between models and language design is especially clear when we analyze, not just deductive power, but expressive power of logical languages. We can think of these languages as designed to describe invariant properties and predicates in one's semantics. One part of this match is the persistence of truth or satisfiability of logical formulas under the relevant invariance relations between models, the other, usually deeper, direction is 'expressive completeness' results showing when the logical language can define all invariant predicates. We will not elaborate this theme, but refer to the model-theoretic literature, cf. Hodges 2020.

Once more, we conclude that in logic, language design and choice of semantics go together.

Caveat: Two senses of representation. The preceding sections moved away from the original issue in this note. We started with a practice of language use and reasoning, and how logic-internal notions represented these external empirical ones. But then we shifted to internal issues such as whether a given logical formula represents its underlying proposition, or whether a concrete model represents the equivalence class defining its structure. We believe that these issues still form a whole by *composing* the two senses of representation. Logical formulas represent natural language sentences, but at the same time they represent logical propositions, so they are a connecting locus between sentences and propositions. And likewise, specific models mediate between actual situations and abstract structure.

This concludes our brief discussion of modeling and representation in logic. What follows is an afterthought questioning the standard 'formal language package' view we took for this.

9 Coda: piecemeal merging versus global juxtaposition

In this paper, we have mainly compared complete natural languages or reasoning practices with the architecture of entire logical languages, semantics, and proof systems for modeling these empirical phenomena. In this final section, we briefly outline an alternative view.

For a start, we could also see the task of logical analysis as *piecemeal problem solving*, which comes with a range of formal solutions as required by the occasion.

Reasoning challenges. Here is a well-known example from the psychology of reasoning:

> Fifteen farmers own at most thirteen cows each. Does it follow that at least two farmers own the same number of cows? (Mercier, Politzer & Sperber 2017)

Experimental subjects turn out to be hard-pressed to justify an answer. What definitely does not work is transcribing the sentence and the putative conclusion into straightforward logical formulas as we suggested above and then applying formal deduction. The key to solving this problem is finding the right way of thinking, or more concretely, a good representation which makes us see the answer in a simple manner. In this particular case, we need to see the problem as an instance of the Pigeon Hole Principle that, if we put k objects into n boxes, where k > n, at least one box will get two objects. In the given case, there are 14 boxes, the number of cows a farmer can own, ranging from 0 to 13, and we place 15 farmers in them.

First representation, then calculus. This example is typical for actual reasoning problems. The difficulty is usually not applying the deduction or computation rules of some calculus, but the prior step of representing the given problem in a way that makes its solution via that calculus easy or at least feasible. And such a representation may work for some but not all problems, so uniform approaches via logical languages and proof systems seem off the mark. What we need then is a repertoire of different representations that help piecemeal with actual scenarios, an ability we can train by just learning and understanding more examples.

Aside: logic and counting. In this piecemeal view, logic still makes sense. Inference patterns codified in logic do occur widely, witness the literature on philosophical or computational logic. But what the preceding example suggests is that reasoning patterns involving counting may be just as basic, a thesis developed in much more detail in van Benthem & Icard 2023. There does not seem to be an obvious priority for logic over arithmetic here.

Hybrids and merges. But there is also a further issue. Problem solving is local in that only a few well-chosen formulas are needed as paraphrases, and only a few relevant inferences need to be drawn. And we do not move entirely into a formal world of derivation and computation with these, leaving the natural language formulation of the problem behind. Indeed, instead of complete and separate natural and formal languages, we can also think of illuminating paraphrases for a problem as *hybrids* of natural language and logical formulas, in the same way as the language of mathematical research is a hybrid of the two.

Dynamic interactions. Well-understood, even the earlier diagram from Moss & Westerståhl 2023 fits this view. While the diagram suggests a strict separation of empirical and logical realms, it can also be seen as making a *methodological distinction.* In reality, there may be a dynamic. A successful formal analysis may influence natural language practice, and some of its notions and notations may make their way into our ordinary linguistic repertoire.

The virtues of hybridity. Perhaps the term 'representation' is then no longer appropriate, as this suggests a separation between what represents and what is represented. The question rather becomes if logic can help *improve* our linguistic and reasoning practices, just as mathematics does. I believe that the looser *hybrid view* is much closer to how logic is used in both mathematical and philosophical practice. The hybrid language of mathematical research and for that matter, of philosophy papers using logic, is a fascinating flexible medium which combines the virtues of both formal and natural languages. The formal components provide precision as needed, but the embedding in natural language makes sure that texts build up interest and *shared purpose.* The natural language also allows for *paraphrasing* and explaining formal proofs at higher less detailed levels increasing our understanding of what makes the formal level tick. I believe that the study of the fascinating mixtures of natural and formal has been neglected in contemporary logic, semantics, and philosophy of language.

10 Conclusion

We have discussed the basic senses in which logic can be said to represent natural language and natural reasoning, involving both syntax and semantics. We then moved to the role of representation inside logic in one picture of semantic invariance and logical language design. We did not present any grand conclusion from all this. Our discussion was rather meant to highlight the variety of representations available in logic, which fits well with the variety of tasks that logic can be applied to. A multi-thread narrative like this seems closer to the realities of applied logic, and if nothing else, it may create awareness of the debatable presuppositions in innocent-looking terminology such as ‘the logical form’ of sentences.

One theme has been ignored in this paper. Logic is not just meant to faithfully *describe* reasoning as humans perform it, it is also a *normative* discipline offering standards. Without the driving force of correction, human intellectual progress would be unimaginable. The latter theme has been ignored in my discussion, but logical representation also has the potential, and perhaps even ambition, to enlighten and where needed, improve practice.

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