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Thought experiments, computer simulations, and real world experiments in scientific knowledge: a comparison

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Abstract. The purpose of this paper is to restate, in a more concise form and taking into account some articles subsequently appeared in the literature, the main point of a previous article regarding the relationship between real world experiments, computer simulations and (empirical) thought experiments. After distinguishing four main families of accounts which have emerged in the literature, it is argued that they, although each contains an element of truth, have failed to distinguish between real experiment, computer simulation and thought experiment. In fact, concerning the empirical intension of the respective concepts, it is a hopeless task to find a qualitative difference which applies exclusively to thought experiments, computer simulations, or real experiments. For every particular characteristic of one of these notions there is a corresponding characteristic in the two others. However, from another point of view, there is between thought experiment and computer simulation on the one hand, and real experiment (or empirical knowledge) on the other, an epistemological-reflective difference which we must not overlook. Unlike computer simulations and thought experiments, real experiments always involve an 'external' or impersonal realisation, namely that of what I propose to call an 'experimental-technical machine', always in causal-real interaction with the experimenter's body.

1 Introduction

The purpose of this paper is to restate, in a more concise form and taking into account some articles subsequently appeared in the literature, a point made elsewhere (cf. Buzzoni 2016) regarding the relationship between real world experiments (hereafter REs), computer simulations (hereafter CSs) and (empirical) thought experiments (hereafter TEs). After distinguishing four main families of accounts which have emerged in the literature, it is argued that they, although each contains an element of truth, have failed to distinguish between RE, CS and TE (Section 2). In the second part of the paper, I shall briefly outline my own account on this topic. To avoid comparisons that are insignificant or of little importance for the philosophy of science, it will be convenient to compare TEs and CSs with real world experiments (hereafter REs). To take the notion of RE as the basis of comparison between CS and TE will enable us not only to better understand the methodological similarities between CS, TE and RE, but also to find a subtle but important distinction between CS and TE. I shall maintain

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that—from the perspective of the analysis of the empirical-methodological intensions of the respective concepts—it is a hopeless task to find a particular methodological trait which applies exclusively to TE, CS, or RE. However, from another point of view, there is between TE and CS on the one hand, and RE on the other an epistemological (or transcendental) difference which we must not overlook. An aspect of the difference between CSs and REs reverberates in the relationship between TE and CS: CSs involve an 'external' realisation, which explains some differences in degree between TEs and CS (for example, the usually greater methodological complexity of the latter).

2 Four families of accounts on the relationship between TE and CS

Two preliminary remarks are in order before we plunge *in medias res*:

1) In first approximation, I shall presuppose the broad sense of CS defined by Winsberg 2013 in its authoritative entry for the Stanford Encyclopedia of Philosophy (now confirmed in the 2019 updated version):

> we can think of computer simulation as a comprehensive method for studying systems. In this broader sense of the term, it refers to an entire process. This process includes choosing a model; finding a way of implementing that model in a form that can be run on a computer; calculating the output of the algorithm; and visualizing and studying the resultant data. The method includes this entire process—used to make inferences about the target system that one tries to model—as well as the procedures used to sanction those inferences. (Winsberg 2013/2019)

2) Here, however, I am interested above all in empirical TEs and CSs, that is, TEs and CSs whose results are liable to correction by new experimental findings. TEs and CSs in formal disciplines deserve a separate treatment, and a fortiori the same applies to philosophical TEs (on these distinct kinds of TE, see Buzzoni 2011, 2022 and 2021).

With this in mind, I propose to distinguish four main families of accounts of the relationship between TE and CS. According to the first view, there is close similarity, or even an identity, of TEs and CSs because "thought experimenting is a form of 'simulative model-based reasoning'" (Nersessian 1992, p. 291; see also Miščević 1992 and 2007, Palmieri 2003, Gendler 2004, Misselhorn 2005, Cooper 2005, Morrison 2009, and Chandrasekharan, Nersessian, and Subramanian 2013). In TEs we manipulate mental models instead of physical models, and we gain knowledge through TEs only to the extent that they contain a manipulation of some mental model.

As far as TEs are concerned (but, *mutatis mutandis*, this also holds true of CSs), the main difficulty with this approach consists in the fact that it seems to assume mental mediators in order to explain how words relate semantically to the world. In both cases, "something must be said about how they acquire their semantical properties." (Häggqvist 1996, p. 81) The upholders of this view have attempted to overcome this problem with the introduction of some "engineering" (Nersessian 2006) or manipulative (cf. Miščević 2007) constraints in their theorizing. But it is only a metaphorical and loose way of speaking: we cannot manipulate mental models in the same way in which we manipulate objects and processes of everyday life.

Moreover, this view has led to treat TEs as a sort of mere provisional means that someday will be abandoned in favour of CSs. The complexity of the natural systems that scientists and engineers are modelling today would be such that the relationship between the different elements of natural systems cannot be captured through TEs, but only by the new computational visualization tools that are being developed in computer science: "computational modeling is largely replacing thought experimenting, and the latter will play only a limited role in future practices of science, especially in the sciences of complex nonlinear, dynamical phenomena." (Chandrasekharan, Nersessian, and Subramanian 2013, p. 239)

As we shall see later, it is true that, generally speaking, there is a difference in degree between CSs and TEs, but, taken as it stands, this claim is a prediction about human knowledge, and therefore it may be considered as a kind of "promissory eliminativism" (in Popper's sense) concerning TEs. As such, it is undermined by Popper's argument according to which, "if there is such a thing as growing human knowledge, then we cannot anticipate to-day what we shall know only tomorrow." (Popper 1957 [1961], italics in original).

The second view to be examined is defended by authors who follow Norton's theory that TEs can be reconstructed as arguments based on both tacit and explicit assumptions (cf., e.g., Norton 1996, pp. 336; see also Norton 1991 and 2004). Following Norton's account, they have drawn a detailed comparison between TE and CS (Stäudner 1998; Stöckler 2000; Velasco 2002; Beisbart 2012, Beisbart and Norton 2012). According to Beisbart, for example, to the crucial question how scientists gain new knowledge, "[t]he argument view answers this question by saying that computer simulations are arguments." (Beisbart 2012, p. 429) Stäudner 1998 made the most detailed comparison between TEs and CSs. As he sums up his results:

> The initial equations that we are striving to solve, together with the relevant boundary values, form a set of 'premises'. The numerical procedure by means of which we calculate the solutions we are looking for corresponds to a 'logical type of inference', that is to a determinate form of argument. The result of the calculation is the 'conclusion'. As in valid arguments true conclusions follow from true premises, we may consider the result of the calculation of a simulation as an adequate

> description of nature if the 'premises' contain adequate descriptions of nature, in the sense that they are empirically confirmed and therefore belong to the well-established 'theoretical patrimony' of the natural sciences. (Stäudner 1998, p. 157; see also p. 8)

The difficulties of this approach are the same as those of Norton's view. For Norton, TEs "are arguments which (i) posit hypothetical or counterfactual states of affairs, and (ii) invoke particulars irrelevant to the generality of the conclusion" (Norton 1991, p. 129; see also Norton 1996, 2004a and 2004b). For Norton, TEs can always be reconstructed as deductive or inductive arguments ("reconstruction thesis") and, more importantly, they must always be evaluated as such:

> The outcome is reliable only insofar as our assumptions are true and the inference valid [...] [W]hen we evaluate thought experiments as epistemological devices, the point is that we should evaluate them as arguments. A good thought experiment is a good argument; a bad thought experiment is a bad argument. (Norton 1996, p. 336)

Many methodological objections can be and have been raised in the literature against Norton's account, but for our purposes the main weakness is that it tends to undermine any distinction between empirical and formal knowledge. Even though in a TE this or that particular empirical element may be "irrelevant to the generality of the conclusion" (for example, in Einstein's lift experiment it is irrelevant whether the observer is or is not a physicist), it is not irrelevant that TEs are generally performed by constructing particular cases, which need concrete elements that are in principle reproducible in specific spatio-temporally individuated situations. TEs, stripped of any reference to concrete experimental situations, are confined to a domain of purely theoretical statements and demonstrative connections. As a result, empirical TEs are reduced to logico-mathematical arguments (on this point, see especially Buzzoni 2008, pp. 67–68 and Stuart 2016).

Now, this same difficulty applies to Beisbart's claim of important features common to CSs and arguments. Given the equation of CS with argument, the same difficulty comes to light that has been noticed in Norton's view about TEs, that is, of reducing empirical TEs to logical arguments. In order to remove this difficulty, in his first papers on CSs Beisbart adopted two strategies. First, he suggested that "running a computer simulation may be thought of as the execution of an argument" (Beisbart 2012, p. 423; this point is interpreted by Beisbart in the light of the extended mind hypothesis of Clark and Chalmers (1998); on this point, as far as CS is concerned, see also Charbonneau 2010). However, this is in contradiction with Beisbart's explicit rejection of the idea that CSs produce new knowledge because they are real world experiments (Beisbart 2012, pp. 425), a thesis which indeed would have undermined his whole argument view. According to the extended mind thesis, cognitive systems may extend beyond a human being. But if the construction of what might be called an 'experimental machine', which extends the original operativity of our organic body, is treated as a part of the mind, there can be no difference in principle between TE and CS on the one hand and RE on the other.

Moreover, Beisbart 2012 accepts the idea that CSs, unlike TEs, are "opaque" and must be explored. This, however, is an autonomous third line of interpretation of the relationships between TE and CS, which we have now to examine separately. As Di Paolo et al. 2000 write:

> A thought experiment has a conclusion that follows logically and clearly, so that the experiment constitutes in itself an *explanation* of its own conclusion and its implications. [...] In contrast, a simulation can be much more powerful and versatile, but at a price. This price is one of *explanatory opacity*: the behaviour of a simulation is not understandable by simple inspection. (Di Paolo et al. 2000, p. 502; cf. also Bedau 1999, Buschlinger 1993, Lenhard & Winsberg 2010).

At least two objections may be raised against this position, and all support the conclusion that opacity must be relativized to a background context and cannot be treated as an absolute concept. First, 'opacity' is no hallmark of CSs (or REs) in contrast with TEs. Indeed, in this regard there is only a difference of degree between CSs and TEs, that is, a difference that may be turned upside down in particular cases: a very simple CS may be less opaque than many TEs (such as Einstein's black body radiation TE: cf. Norton 1991). Moreover, this thesis presupposes that TEs have a kind of almost Cartesian clearness, which, at least apparently, like that of Descartes' *cogito*, would be static and without a history. This presupposition has been probably inspired by Hacking's claim that, while REs "have a life of their own", TEs "are rather fixed, largely immutable". But this thesis is untenable. To see this, it is sufficient to recall the history of the interpretations of the most important TEs (such as Maxwell's Demon or Galileo's falling bodies).

The second objection is even more serious, since it concerns a fundamental trait of scientific thinking. A particular truth-claim resulting from a CS may be considered as scientific only under the condition that it is in principle intersubjectively testable. CSs must consist in concrete methodical procedures which we may, at least in principle, reconstruct, re-appropriate and evaluate in the first person. No matter how complicated the 'modellization' or even 'mechanization' of cognitive performances may be, if we accept the results of a CS, we presuppose that any change concerning the hardware/software may be in principle reconstructed and reappropriated in the first person (this is also true of a random number generation) (for this objection, Buzzoni 2008 and 2016).

According to the fourth view, CSs are considered as intimately connected with real experiments. Recently, the relationship between CS and traditional experimentation has attracted more and more attention (cf. Galison 1996; Keller 2003; Parker 2010; Morrison 2009; Chandrasekharan, Nersessian, and Subramanian 2013; Guala 2002, 2005; Morgan 2002, 2005; Norton and Suppe 2001; Winsberg 2003 [2010]; Küppers and Lenhard 2005a; and 2005b, Lenhard 2007). Among these authors, the thesis most frequently recurring is that a CS is, as Winsberg has called it, a "hybrid of experiment and theory" (Winsberg 2003 [2010], p. 220. For a similar view, see Norton and Suppe 2001; Guala 2002, 2005; Morgan 2002, 2005; Küppers and Lenhard 2005a—who speak of a "quasi-empirical character" of CSs—and 2005b; Lenhard 2007). Other authors have emphasized the experimental aspects of CSs to such an extent that the latter are considered as falling under the more general concept of experiment (cf. esp. Morrison 2009 and Norton and Suppe 2001). As Norton and Suppe 2001 write:

> Simulations often are alleged to be only heuristic or ersatz substitutes for real experimentation and observation. This will be shown false. Properly deployed simulation models are scientific instruments that can be used to probe real-world systems. Thus, simulation models are just another source of empirical data. (Norton and Suppe 2001, p. 87)

We can, of course, undertake no minute discussion of the many varieties of this approach. I shall confine myself to criticising the claim that CSs provide knowledge in the same way as that in which experiments do. It was rightly noted that a computer simulation may give us information about the actual world, only because "we have independent evidence of the model's significance": "we will know whether or not the theory of cosmic defects is adequate, not via computer experiments, but through the use of satellite-based instruments." (Hughes 1999, p. 142; a similar objection has been made, among others, by Muldoon 2007, p. 882; Frigg & Reiss 2009; Beisbart 2012, p. 245).

But this objection should be formulated in a more radical form, by saying that in a RE the construction of an experimental setup that extends the original operativity of our organic body is connected not only to the "method of variation"—as emphasized by Mach—, but also to the causal interaction between our organic body and the 'experimental machine' that actually makes up a scientific experiment. On the contrary, in the case of CS, our 'contact' with reality is always mediated by models, to which real objects may or may not correspond (Buzzoni 2008).

One might try to elude this difficulty, as Lusk 2016 did, by maintaining that "insofar as certain common forms of measurement interact with their target and return new knowledge of their target system, simulations, under certain conditions, can as well." (p. 145) But this is not the point: the point is that in the case of CSs we cannot causally interact with the intended target in the same causal sense in which we interact with real word objects by experimenting, that is, by means of our organic body.

This point applies as much to the account of Lusk (2016) as it does to those of Johannes Lenhard and Claus Beisbart. According to Johannes Lenhard, the process of simulation modelling

> takes the form of an explorative cooperation between experimenting and modeling and that it is this characteristic mode of modeling that turns simulations into autonomous mediators in a specific way; namely, it makes it possible for the phenomena and the data to exert a direct influence on the model. (Lenhard 2007, pp. 176–177)

Although not citing Lenhard, this is also the view that Beisbart, clearly changing his mind, has developed since 2018. Now Beisbart argues that CSs, while not essentially arguments (as previously argued), bear many similarities to real-world experiments, while not identifying with them. This is possible because, although they are not REs, they "can model possible experiments and do often do so. Using this suggestion, we can account for the similarities between experiments and CSs without unduly assimilating the two methods." (Beisbart 2018, p. 173)

In this way the author rightly distinguishes CSs and REs, but he is no longer able to distinguish CSs and empirical TEs. He might reply that, even though he admits that intervention and observation can be modeled in a CS *study*, this happens in a different way:

> In a CS study, the simulationalist can set the initial conditions and the values of important parameters, and this is in fact what is often done. This is similar to manipulation and activities of control on the part of the experimenter in an experiment. [...] In some cases, the simulationalist may even consciously imitate the activities typical of an experimenter. We can thus say that simulation scientists can make quasi-interventions that reflect possible interventions in experiments. (Beisbart 2018, p. 194)

However, this is by no means sufficient to distinguish empirical CSs not only, as already mentioned, from REs, but also from empirical TEs, which are empirical only insofar as they contain explicit or implicit reference to a set-up that is in principle, first, realizable and, second, capable of entering into causal interaction with our body. Thus, when he speaks of "quasi-intervention" in a simulation in order to express with greater accuracy his point of view, he reveals de facto his difficulty in distinguishing between CSs and TEs of the empirical type. And the same applies to the claim that the model targeted in a CS is "an imagined experiment," an expression by which Beisbart would like to distinguish the conceptual content of CSs from that of TEs, but which, by a singular irony, was used many times in the past precisely as a synonym for "thought experiment."

It is therefore no accident that many statements by Beisbart are reminiscent of similar statements made by this or that author about empirical TEs: "Other simulations assume that the laws of nature are different from those in this world. [...] Setting the initial conditions and tinkering around with several parameters can nevertheless be conceptualized as a surrogate for an intervention, if only one that is not physically possible to us, but which would be of interest." (Beisbart 2018, pp. 196–197)

In sum, Beisbart is able to distinguish CSs and real world experiments only at the cost of confusing CSs and TEs. This is not only contrary to his explicit intent to distinguish CSs from both TEs and REs, but more importantly it fails to take into account the essential epistemological difference between CSs and REs that we have already pointed out and that also undermines Lenhard's sophisticated analyses: in every CS the real interaction between the experimenter's body and empirical reality is lost, at least in its operational sense.

I shall return to the importance of this point later in order to consistently conceive of a relationship of unity and distinction between CSs and REs on the one hand and CS and TE on the other.

3 Computer simulations and thought experiments vs. real world experiments

In the second part of this paper, I shall briefly outline an account of CSs as compared with TEs that manages to avoid at least some of the difficulties we have just considered.

According to Mach, the principle of economy is not only the source of science as such—and hence of REs—, but also of thought experimentation: We experiment with thought, so to say, at a low price because our own ideas are more easily and readily at our disposal than physical facts (Mach 1905a, p. 183–184, Engl. Transl., pp. 136–137). Moreover, both real world experiments and TEs are based on the "method of variation" (*Methode der Variation*): while in REs it is natural circumstances, in TEs it is representations that are made to vary in order to see the consequences of those variations (cf. Mach 1905a, 1905b, 1905c, 1883).

Now such similarities between TEs and REs may be easily extended to include CSs: on the one hand, historically speaking, CSs also aroused out 'economical' reasons in the broad sense in which the term was used by Mach (cf. Keller 2003); on the other hand, it is difficult to deny that CSs are also based on the "method of variation".

But it is very easy to find many other similarities. For instance: 1) TEs, CSs and REs are constituted by a theory and a particular, well-specified experimental situation (Buzzoni 2013, pp. 97–98); 2) all of them ask questions about nature and its laws in a theory-laden and idealized way, so that the meaning of all of them must always be interpreted; 3) in all cases visualisation, perspicuity, intuitive appeal, and clarity are important because TEs, CSs and REs apply general hypotheses to particular cases that are relevant for testing their truth or falsity (for the importance of visualisation in CSs, see for example Winsberg 2003 [2010] and Beisbart 2012).

For this reason, there is a prima facie ground for maintaining a much more radical thesis. We shall argue that it is no coincidence that we find so many similarities between REs, TEs, and CSs in the literature, since these similarities can in principle be multiplied without limit. From the perspective of the analysis of the empirical intensions of the respective concepts, REs, TEs, and CSs show only differences in degree, not in kind.

In order not only to justify, but also to restrict the meaning of this thesis, it will be best to discuss a point of contact between TE and CS that I have already mentioned. As Mach pointed out, when faced with the slightest doubt about the conclusions of a TE, we have to resort to REs:

> The outcome of a thought experiment [...] can be so definite and decisive that any further test by means of a physical experiment, whether rightly or wrongly, may seem unnecessary to the author. [...] The more uncertain and more indefinite the outcome is, however, the more the thought experiment pushes towards the *physical experiment* as its natural continuation, which must now intervene to complete and determine it. (Mach 1905a, pp. 185, Engl. Transl., pp. 137–138; italics restored and translation modified)

It is true that TEs and CSs have a certain autonomy as regards experience in the sense that both anticipate an answer to a theoretical problem without resorting directly to REs. Empirical TEs and CSs anticipate, at the linguistic-theoretical or representational level, a hypothetical experimental situation so that, on the basis of previous knowledge, we are confident that certain interventions on some variables will modify some other variables, with such a degree of probability that the actual execution of a corresponding real world experiment becomes superfluous.

But Mach was right, since this autonomy is only a relative one. If someone puts two coins, and then two more coins into an empty money box, I know that there are now four coins in that money box, and I will persist in that knowledge even if, say, the money box immediately afterwards falls into a deep lake so that I will never again be able to count how many coins it contains. But this knowledge can never outstrip our initial knowledge as to its certainty or degree of justification: for example, if the person that put the coins into the money box was a conjurer, this might cause doubts about the box's content that could be dispelled only by resorting to experience.

Similarly, if in the simulation of a hurricane there appeared objects that my background knowledge told me should not appear, I might be faced by a difficulty that only a real test, in the last analysis, could solve in the most reliable way.

The just mentioned difference between TEs and CSs on the one hand and REs on the other, is a very important exception to the rule that, from the perspective of the analysis of the empirical-methodological intensions of the respective concepts, REs, TEs, and CSs do not essentially differ. But strictly speaking this is no exception because it expresses not an empirical, but an epistemological or reflective-transcendental difference between TE and CS on the one hand, and REs on the other.

More precisely, this epistemological-transcendental difference has two distinct, but related, sides or senses, one subjective and one objective. The subjective side consists in the capacity of the mind to anticipate a hypothetical or counterfactual experimental situation. From this point of view, what TEs and CSs have over and above real ones is only the fact that they exist in a purely hypothetical sphere. But this transcendental difference has also an objective counterpart: what REs have over and above TEs and CSs is only the fact that they are the expression of causal-operational interactions between our bodies and the surrounding reality.

In this connection, Kant's example of a hundred dollars is very instructive. On the one hand, "the real contains no more than the merely possible. A hundred real thalers do not contain the least coin more than a hundred possible thalers." On the other hand, "My financial position is, however, affected very differently by a hundred real thalers than it is by the mere concept of them (that is, of their possibility). For the object, as it actually exists, is not analytically contained in my concept, but is added to my concept (which is a determination of my state) synthetically" (KrV B 627, AA III 401).

It is interesting to note that the epistemological-transcendental difference between TE and CS on the one hand and RE on the other is the true reason of the fact that the intensions of the concepts of TEs, CSs, and REs coincide, as do the hundred real dollars and the hundred merely thought ones. Every (empirical) TE or CS corresponds to a real one that satisfies the same conceptual characteristics, and vice versa. *All REs may also be thought of as realisations of TEs or CSs; conversely, all empirical TEs and CSs must be conceivable as preparing and anticipating RE: They must, that is, anticipate a connection between objects which, when thought of as realised, makes TE and CS coincide completely with the corresponding RE.*

As we shall see now, an aspect of this last difference reverberates in the relationship between TE and CS. Briefly stated: any simulation, even a computer one, involves a kind of *real* execution, one that is not merely psychological or conceptual. In TEs the subject uses in the first person concepts, inferences, etc.; in contrast, REs and CSs involve, in a very particular sense, an 'external' realisation, so that we can reconstruct them only *ex post* (reconstructed *ex ante*, they are TEs again!). In a CS, the striking of certain keys is followed by a sequence of actual physical steps, i.e., the operations carried out by the hardware and the software, with the appearance of certain signs on the screen or in the print-out. As in REs (though in a distinct sense), this execution depends on us for its realisation only in the initial moment when we set off its 'mechanism'. The initial action is followed by a real process that occurs independently of a perceiving mind and ends, for example, with a pointer moving on a dial.

Thus, CS has two distinct aspects: on the one hand, as TE does, it anticipates an answer to a theoretical problem without resorting directly to experience. On the other hand, the similarities between the two should not obscure the distinction between the *hypothetical-counterfactual* context where the test of a hypothesis is planned, and the *real* context where this plan is actually carried out. CSs share the first aspect with TEs, and the second with real ones. A plan for testing the relevant hypothesis must have been devised before CSs get under way (this holds also for "experimental simulations", such as that of a car prototype in a wind tunnel). But CSs involve an application of logics and mathematics to reality which is, in the last analysis, a technical-practical execution.

From this point of view, we may recognize certain elements of truth in Di Paolo et al. 2000's opacity thesis, in Fritz Rohrlich's claim that CS provides a new and different methodology for the physical sciences (Rohrlich 1990), and finally in Lenhard's thesis that "while thought experiments are a cognitive process that employs intuition, simulation experiments rest on automated iterations of formal algorithms." (Lenhard 2018, p. 484; cf. also Roman Frigg and Julian Reiss 2009). The realisation involved in a CS is different in meaning from the causal interactions occurring in REs: as Hughes aptly says, when physicists talk of 'running experiments on the computer', they presumably do not mean that CSs are performed to learn something about computers. But this suggests at least one of the reasons for the *de facto* greater methodological complexity of CSs in comparison with TEs. Accuracy, error analysis, calibration, and in general the management of uncertainty, though not peculiar to CSs, are *de facto* concepts that we encounter more frequently in discussing CSs than TEs (cp. above all Winsberg 2003 [2010], and Muldoon 2007).

4 Conclusion

The main conclusions, at which we have arrived so far, may be briefly summed up as follows:

1) The attempts to find a distinction in logical kind between TEs, CSs and REs from an empirical-operational or methodological sense break down: for every particular characteristic of one of these notions there is a corresponding characteristic in the others.

2) There is a difference in kind (an epistemological-reflective difference) between TEs and CSs on the one hand and REs on the other (which, on reflection, is the deepest reason of their similarities!).

3) An aspect of this last difference reverberates in the relationship between TE and CS. CSs involve an 'external' realisation, which must be carefully distinguished from that involved in REs, since CSs are not performed to learn something about computers.

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